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perioduli struqturebis eleqtrodinamikuri Tvisebebis Seswavla zogierTi kompleqsuri masalebis Tvisebebis misaRwevad

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problemis aqtualoba. Tanamedrove cifruli da analoguri eleqtronuli mowyobilobebi (kompiuterebi, mobiluri kavSirgabmulobis mowyobilobebi da a. S.) mniSvnelovan rols asruleben sazogadoebriv cxovrebaSi. maT gareSe warmoudgenelia Tanamedrove medicinis, sakomunikacio sistemebis, sabanko seqtoris da sxva dargebis ganviTareba.

cifruli da analoguri sistemebis ganviTarebis tendenciebi miuTiTebs imaze, rom maTi momavali samuSao sixSireebis diapazoni ufro da ufro mikrotalRur (terahrcul da infrawiTel) areSi gadaiwevs. am diapazonSi standartuli masalebi (naxevargamtarebi da metaluri zedapirebi) gansxvavebul Tvisebebs avlenen, kerZod isini metad STanTqaven am sixSiris talRebs an xdebian rTulad dasamuSavebeli. amis gamo maTi gamoyeneba mniSvnelovnad SezRudulia. amitom aqtualuri gaxda axali masalebis, struqturebis da midgomebis Zieba. am masalebs zogadad kompleqsuri masalebi da aseve metamasalebi ewodebaT.

rogorc CvenTvis cnobilia, istoriulad kompleqsur masalebze moTxovnileba egreT wodebuli “stelsis” amocanidan daiwyo, rodesac miznad iyo dasaxuli Seqmniliyo dafena minimaluri gabnevis ganivkveTiT, ris Sedegadac igi uCinars gaxdida radarebisTvis samxedro TviTmfrinavebs.

kompleqsuri masalebi, zogadad, warmoadgenen xelovnur nivTierebebs, romelTa eleqtromagnituri Tvisebebi gansxvavdeba maTi Semadgeneli nivTierebebis eleqtromagnituri Tvisebebisgan.

kompleqsuri masalebis bazaze, kerZod, SesaZlebelia damzaddes aRniSnul sixSirul diapazonSi momuSave logikuri da integraluri sqemebi, aseve sxvadasxva eleqtronuli mowyobilobebi, magaliTad: sixSiruli filtri, cirkulatori, simZlavreTa gamyofi, simZlavreTa Semrevi, talRgamtari, aseve antenuri struqturebi, rogoricaa mimarTuli gamosxivebis mqone antena egreTwodebuli fazirebuli antena romlis gamosxivebis mimarTuleba SegviZlia vcvaloT eleqtronulad. metamasalebis gamoyeneba am mizniT dRes dReobiT metad aqtualuria, radgan Tanamedrove eleqtronuli mowyobilobebis gadasvla maRal sixSirul diapazonSi arsebuli Zveli teqnologiebis bazaze rTulia da zog SemTxvevaSi SeuZlebeli maTi zomebis Semcirebis gamo. Tanamedrove teqnologiis gamoyenebiT kompleqsuri masalebis damzadeba SedarebiT martivia da is ar aris SezRuduli sixSiruli diapazoniT.

unda aRiniSnos, rom arsebobs bunebrivi kompleqsuri masalebi. magaliTad, bunebrivi kiraluri garemo, romelic iyo cnobili jer kidev XIX saukunis dasawyisidan. termini “kiraluri” pirvelad gamoyenebuli iqna uiliam tomsonis mier da niSnavs obieqtis SeuTavseblobas mis sarkul anareklTan - aranairi brunviTi da gadataniTi moZraobebiT. bunebriv kiralur obieqtebs warmoadgenen Saqris, aminomJavebis, dnm-s da organuli polimerebis molekulebi (http://www.complex.mat.ethz.ch). rogorc wesi, bunebriv kompleqsur masalebs ar gaaCniaT sasurvel sixSirul diapazonSi praqtikisaTvis saWiro Tvisebebi. amis gamo, zogadad, maTi gamoyeneba praqtikaSi SeuZlebelia. amitomac arsebobs maTi xelovnurad miRebis didi moTxovnileba.

zogadad cnobilia, rom xelovnuri kompleqsuri masala SeiZleba Seiqmnas Tu Cveulebriv dieleqtrikSi SeviyvanT gamtari elementebisagan Semdgar samganzomilebian periodul mesers [1-6].

TanamedrovE teqnologiis ganviTarebiT SesaZlo gaxda kompleqsuri masalebis damzadeba. unda aRiniSnos, rom aseTi teqnologiebi saqarTveloSic arsebobs. italielebTan, amerikelebTan da espanelebTan TanamSromlobiT, mimdinareobs Txevadi kristalebis eleqtromagnituri Tvisebebis gamokvleva optikur diapazonSi, realuri eqsperimentebis saSualebiT [7-16]. kerZod, interess warmoadgens optikur diapazonSi momuSave gadawyobadi organzomilebiani da samganzomilebiani fotonuri kristalebis miReba Txevadi kristalebis bazaze.

rogorc cnobilia, fotonuri kristali warmoadgens periodul mesers, romlis periodi talRis sigrZis rigisaa. eleqtromagnituri talRis gavrceleba aseT kristalSi Seesabameba eleqtronis gavrcelebas naxevargamtarSi. maqsvelis gantolebebis amonaxsni fotonur kristalebisaTvis aCvenebs rom arsebobs talRis sigrZis iseTi mniSvnelobebi, rodesac maTSi talRis gavrceleba ar xdeba. gadawyobadi fotonuri kristali warmoadgens iseT fotonur kristals, romlis konfiguracia SeiZleba Seicvalos gare eleqtromagnituri velis zemoqmedebiT. aseTi saxis fotonuri kristalebi did gamoyenebas pouloben optikuri kavSirgabmulobis sistemebSi.

kvlevis obieqti da amocanebi. rogorc cnobilia, standatruli masalebi, rogoricaa gamtarebi, dieleqtrikebi da naxevradgamtarebi, aRiwerebian zogadad ori parametris saSualebiT: dieleqtrikuli ε da magnituri μ SeRwevadobebiT.

erTgvarovan izotropul dieleqtrikis SemTxvevaSi am parametrebs gaaCniaT namdvili mniSvnelobebi. maT SeiZleba gaaCndeT aseve kompleqsuri mniSvnelobebi, rasac aqvs adgili, magaliTad danakargebis mqone garemoSi. zogadad, anizotropuli garemos SemTxvevaSi, dieleqtrikuli da magnituri SeRwevadobebi warmoadgenen tenzorul sidideebs, magaliTad plazmis da damagnitebuli feritis SemTxvevaSi. aseT garemoSi kavSiri induqciis veqtorebsa da daZabulobis veqtorebs Soris Semdegi saxiT Caiwereba:

$$D_i = \varepsilon_{ij} E_j , \;\; B_i = \mu_{ij} H_j .$$

CamoTvlili masalebisagan gansxvavebiT kompleqsuri masalebis aRwera moiTxovs kidev damatebiT or α, β parametrs, romelTac admitansebi ewodebaT. gasagebia zogadad, rom am oTxive parametrs aseve tenzoruli buneba SeiZleba gaaCndeT. aseT rTul garemos bianizotropuli kompleqsuri garemo ewodeba. am SemTxvevaSi zemoaRniSnuli kavSiri gamoisaxeba rogorc

$$D_i = \varepsilon_{ij} E_j + \alpha_{ij} H_j , \;\; B_i = \mu_{ij} H_j + \beta_{ij} E_j .$$

kerZo SemTxvevas warmoadgens biizotropuli garemo, romelSic oTxive parametrs namdvili mniSvnelobebi gaaCniaT. am damatebiT admitansebzea damokidebuli is saintereso Tvisebebi, romelnic kompleqsur masalebs gaaCniaT.

warmodgenil naSromSi ganixileba, droSi harmoniuli eleqtromagnituri talRis difraqciisa da gabnevis amocanebi zogierT metalo - dieleqtrikul struqturebze. ganixileba aseTi struqturebis sasruli da usasrulo SemTxvevebi. Zogadad, struqtura warmoadgens periodul mesers, romelic moTavsebulia dieleqtrikSi. meseri Sedgeba mcire eleqtruli zomebis mqone gamtar elementebisagan, romlebsac garkveul sixSireebze rezonansuli Tvisebebi gaaCniaT. Cveni amocanaa, aseTi sistemebis eleqtrodinamikuri Tvisebebis Seswavla rezonansul sixSireebis areSi, rodesac isini kompleqsuri garemos Tvisebebs iZenen.

cnobilia, rom yovel rezonansul sixSireze TviToeul elementze aRZruli denis amplituda mkveTrad izrdeba. dasawyisSi, maRali vargisianobis Sedegad rezonansul sixSireze, sistema iZens dacemuli talRis energiis did nawils da procesis damyarebis Semdeg srulad gadasxivdeba. am dros, sixSiris zrdisas, yovel ganmxoloebul elementsac ki uCdeba axali miulevadi speqtraluri komponenti, rac aCens Sori zonis diagramaSi axal foTols. rodesac gvaqvs aseTi elementebis erToblioba meserSi, maTi eleqtrodinamikuri urTierTqmedeba ansamblSi izrdeba da iwvevs denis kidev ufro met gazrdas yovel elementze. aRniSnuli urTierTqmedeba damokidebulia agreTve elementebs Soris manZilze, radgan igi gansazRvravs urTierTqmedebis energiis fazas. elementebs Soris manZili aseve SeiZleba iyos rezonansuli. aseT movlenas SeiZleba vuwodoT ormagi rezonansi. rodesac sistema moTavsebulia dieleqtrikSi, moiZebneba sruli sistemis iseTi parametrebi, rodesac aRniSnuli efeqtebi mkveTrad izrdeba da am dros igi iZens kompleqsuri garemos Tvisebebs.

mesris elementis geometriuli forma gansazRvravs, romel tipis kompleqsur garemos Seesabameba aRebuli struqtura mis rezonansul sixSireebze. kerZod, marjvena an marcxena kiralobis mqone elementebis struqtura dieleqtrikSi Seesabameba kiralur garemos; Tu mesris elementi warmoadgens koncentrirebul or Ria rgols, maSin garkveul sixSireebze Sesabamis garemos uaryofiTi gardatexis maCvenebeli gaaCnia da a. S.

zogadad, metad sasurvelia naSromSi ganxiluli struqturebis siRrmiseuli Seswavla im mizniT, rom SevZloT imis dadgena, Tu konkretulad romel kompleqsur garemos Seesabamebian isini da risi tolia maTi eleqtrodinamikuri parametrebis (SeRwevadobebis da admitansebis) mniSvnelobebi. es mogvcems saSualebas SemdgomSi amovxsnaT Sebrunebuli amocana da kompiuteruli modelirebis saSualebiT davamodeliroT sasurveli parametrebis da Tvisebebis mqone kompleqsuri masala. unda aRiniSnos, rom aseTi amocanis gadaWra dResdReobiT metad rTul problemas warmoadgens. saqme imaSia, rom elementebis perioduli wyoba dieleqtrikSi qmnis miRebuli struqturis anizotropias, ris gamoc misi oTxive eleqtrodinamikuri parametri tenzorul xasiaTs iZens. Sesabamisad izrdeba ucnobi parametrebis raodenoba, romelic am SemTxvevaSi, zogadad aris $4\times3\times3=36$. gamosavali aseT rTul SemTxvevidan SeiZleba iyos, Tu davarRvevT elementebis periodulobas dieleqtrikSi, ise rom yovel maTgans damoukidebeli orientacia da mdebareoba gaaCndes. maSin maTi raodenobis gazrdis Sedegad SesaZloa saZiebeli eleqtrodinamikuri parametrebis gasaSvaloeba. zogadad damtkicebulia, rom aseTi saxis struqtura unda Seesabamebodes biizotropul kompleqsur garemos, romelSic kavSiri induqciis da daZabulobis veqtorebs Soris SedarebiT ufro martivad gamoisaxeba:

$$\vec{D} = (\varepsilon + \mu\alpha\beta)\vec{E} + i\mu\alpha\vec{H}, \quad \vec{B} = -i\mu\beta\vec{E} + \mu\vec{H}.$$

kerZo SemTxvevaSi, Tu sruldeba piroba $\alpha = -\beta$ maSin garemos telegenis garemo ewodeba. Tu $\alpha = \beta \neq 0$, maSin garemos kiraluri Tvisebebi gaaCnia.

miuxedavad aseTi gamartivebisa, am SemTxvevaSi Cndeba axali problema. saqme imaSia, rom elementebis raodenobis aseTi gazrda iTxovs ricxviTi gamoTvlebis dros metismetad did kompiuterul resursebs.

kvlevis mizani. kompleqsuri masalebis eleqtrodinamikis ZiriTadi mizani, praqtikis TvalsazrisiT, mdgomareobs:

1. xelovnuri kompleqsuri masalebis Tvisebebis SeswavlaSi, maTi praqtikaSi gamoyenebis mizniT.

2. imis dadgenaSi Tu ramdenad SesaZlebelia sasurveli Tvisebebis mqone kompleqsuri masalebis damzadeba, romelnic imuSaveben sasurvel sixSirul diapazonSi.

mocemuli sadisertacio naSromi wamroadgens erTerT etaps am zogadi miznebis misaRwevad. masSi ganixileba ramodenime saxis metalo-dieleqtrikuli struqtura da xdeba maTi Teoriuli analizi. Semdeg, kompiuteruli modelirebis saSualebiT xdeba maTi Tvisebebis (kiraloba, uaryofiTi gardatexa da a. S.) gamokvleva.

kompiuteruli modelireba da ricxviTi eqsperimentebi win unda uZRvodes realur eqsperiments, sistemis optimaluri parametrebis dasadgenad. amasTanave aseTi ricxviTi eqsperimenti aris bevrad ufro advili, moxerxebuli da ar aris dakavSirebuli did xarjebTan.

unda aRiniSnos rom am mimarTulebiT muSaobisas miRebuli Sedegebi aprobaciis mizniT wardgenil iqna samsjelod konferenciebze MMET 2010, DIPED 2012 da maT dadebiTi Sefaseba daimsaxures. amis Semdeg aRniSnuli Sromebi miRebul iqna dasabeWdad JurnalSi "Journal of Communications Technology and Electronics" da aseve saberZneTSi, JurnalSi "Journal of Applied Electromagnetism".

problemis Tanamedrove mdgomareoba. 1967 wels viqtor veselagom gamoaqveyna Sroma, romelSic aRniSna, rom Tu garemos eleqtrul da magnitur SeRwevadobebs gaaCniaT uaryofiTi mniSvnelobebi, maSin aseT nivTierebas eqneba uaryofiTi gardatexis maCvenebli [17]. marTlac, Tu ganvmartavT gardatexis koeficients rogorc $n = \sqrt{\varepsilon}\sqrt{\mu}$ da davuSvebT, rom $\varepsilon = -\varepsilon'$, $\mu = -\mu'$, maSin miviRebT $n = \sqrt{-\varepsilon'}\sqrt{-\mu'} = i^2\sqrt{\varepsilon'}\sqrt{\mu'} = -n'$. aseT garemoSi talRis eleqtruli, magnituri da gavrcelebis mimarTulebis veqtorebi qmnian marcxena brunvis sistemas. pirveli uaryofiT indeqsiani nivTiereba Seiqmna 30 wliT gvian, mas Semdeg rac smitma Seqmna rezonatoruli meseri.

dResdReobiT radiofizikaSi sul ufro da ufro did interess iwvevs aseTi struqturebis da zogadad, kompleqsuri masalebis gamoyeneba, radgan maTi saSualebiT SeiZleba unikaluri Tvisebebis mqone sistemebis Seqmna. kerZod, did interess iwvevs iseTi sistemebis Seqmna romelnic iyeneben bianizotropuli da kiraluri garemoebis Tvisebebs. kiraluri masalebis Teoriuli gamokvleva maTematikuri fizikis meTodebiT warmoadgens metad mniSvnelovan da saintereso amocanas. SeiZleba gamoyofil iqnas aseTi amocanebis ori klasi: 1. speqtraluri amocanebi, rodesac gamosakvlevia kiraluri masalebis bazaze miRebuli rezonatoruli sistemebi. 2. aRznebis sawyis-sasazRvro amocanebi, romlebSic Seiswavleba kiraluri masalebis bazaze Seqmnili sxvadasxva talRgamtari sistemebis aRznebis procesebi da eleqtromagnituri talRebis gavrceleba aseT sistemebSi.

zogadad, kompleqsuri masalebis eleqtrodinamikaSi SeiZleba gamoyofil iqnas ori ZiriTadi amocana: 1. Sebrunebuli amocana, rodesac moiTxoveba xelovnurad iqnas miRebuli sasurveli eleqtrodinamikuri parametrebis (SeRwevadobebis da admitansebis) mqone struqtura. 2. pirdapiri amocana, romelic mdgomareobs imaSi, rom struqturaze difraqciis da gabnevis amocanis amoxsnis safuZvelze dadgindes misi, rogorc kompleqsuri masalis eleqtromagnituri Tvisebebi (kiraloba, uaryofiTi gardatexa da a. S.).

CvenTvis cnobilia msoflioSi ramodenime jgufi, romelnic atareben Teoriul da eqsperimentalur kvlevebs ganxilul dargSi. kerZod, arsebobs aseTi jgufi fineTSi - helsinkis teqnologiur univesitetSi, kanadaSi [2], belorusiaSi [3], SveicariaSi, SvedeTSi [4], ruseTSi - peterburgis teqnikur universitetSi, moskovSi, radioteqnikis da eleqtronikis institutSi [18, 19], aSS - pensilvaniis universitetSi [1, 20-39], arsebobs aseve jgufebi inglisSi, safrangeTSi, da germaniaSi.

miuxedavad imisa, rom kompleqsuri masalebis gamokvleva gamoyenebiT eleqtrodinamikaSi ukve aTwleuls aRwevs, dRemde ar arsebobs dasrulebuli Sesabamisi analitikuri Teoria, ris gamoc aseTi struqturebis gamokvleva xdeba ricxviTi meTodebiT gamoTvliTi fizikis da kompiuteruli modelirebis saSualebiT.

Catarebuli kvlevis siaxle. sadisertacio naSromis siaxles warmoadgens:

1. damxmare gamomsxiveblebis meTodis ganviTareba da misi gamoyeneba sxvadasxva metalo-dieleqtrikuli struqturebis gamosakvlevad. usasrulo perioduli struqturebis SemTxvevaSi miRebulia perioduli grinis funqcia, romelic am SemTxvevaSi damxmare gamomsxivebelis velis rols asrulebs.

2. Seqmnili programuli paketi, romelSic arsebobs ramodenime saxis mesris elementis da dieleqtrikis formis SerCevis saSualeba. programaSi arsebobs SesaZlebloba vcvaloT struqturis rogorc geometriuli parametrebi, aseve dieleqtrikis SeRwevadobebi. amasTanave, paralelurad mowmdeba TviT algoriTmis sizuste. maSasadame SesaZloa sakmaod farTo klasis metalo-dieleqtrikuli struqturebis modelis Seqmna da maTi testireba winaswar arCeuli sizustiT.

Seqmnili programuli paketi warmoadgens rogorc saswavlo, aseve samecniero saSualebas. rogorc saswavlo saSualeba, igi xels Seuwyobs aRniSnuli TematikiT dainteresebul studentebs intuiciis gamomuSavebaSi, codnis miRebasa da ganmtkicebaSi. rogorc samecniero saSualeba, igi daexmareba am dargSi momuSave yvela mecniers metalo-dieleqtrikuli struqturebis Tvisebebis SeswavlaSi da maTi parametrebis optimizaciSi. amasTanave, igi SeiZleba wardgenil iqnas sxva universitetebSi da samecniero kvleviT institutebSi

3. kasinis elementis ganxilva. naSromSi Seswavlilia eqvsi gansxvavebuli formis mesris elementi. esenia: gamtari monakveTi, gamtari Ria rgoli, ori gamtari koncentrirebuli Ria rgoli, gamtari spirali, gamtari Ω- elementi da gamtari kasinis elementi. naSromSi [30] ganxilul iqna talRgamtaruli amocana, rodesac talRgamtaris ganivi kveTi kasinis ovals warmoadgens. kasinis elementis ganxilva kompleqsuri garemos modelirebis mizniT, warmoadgens sadisertacio naSromis erTerT siaxles, radgan rogorc CvenTvis cnobilia, igi am mizniT araa gamoyenebuli sxva mkvlevarTa mier. am elementis gamokvlevam gviCvena, rom mas gaaCnia rezonansuli Tvisebebi farTo sixSirul diapazonSi da amitom mis bazaze miRebul struqturas aseve unda gaaCndes kompleqsuri Tvisebebi sixSireebis farTo areSi.

kvlevis ZiriTadi ricxviTi meTodi. maTematikuri fizikis mravali amocana daiyvaneba wrfiv araerTgvarovan diferencialur gantolebamde kerZo warmoebulebSi. aseTi gantoleba zogadi operatoruli saxiT Caiwereba rogorc

$$\hat{L}f(x) = g(x),$$

sadac $\hat{L}$ warmoadgens wrfiv operators, xolo $g(x)$ - cnobil funqcias. fizikurad, es gantoleba Seesabameba ucnobi $f(x)$ velis povnas, rodesac cnobilia misi wyaroebis $g(x)$ ganawileba raime (D) areSi.

rogorc cnobilia, am gantolebis zogadi amonaxsni SeiZleba gamosaxul iqnas Sesabamisi grinis funqciis saSualebiT, romelic akmayofilebs gantolebas

$$\hat{L}G(x,y) = \delta(x-y),$$

sadac δ delta funqciaa. grinis funqcia Tavisi fizikuri azriT aris wertilovani wyaros mier Seqmnili veli. sawyisi gantolebis aRniSnuli amonaxsni Caiwereba rogorc

$$f(x) = \int_{(D)} G(x,y)\, g(y)\, dy.$$

grinis funqciis konkretuli saxe damokidebulia sasazRvro da aseve damatebiT pirobebze. eleqtrodinamikis, hidroaerodinamikis, drekadobis Teoriis da fizikis sxva amocanebSi ucnobia pirvel rigSi TviT wyaroebis $g(x)$ ganawileba. es ucnobi ganawileba SeiZleba gamowveul iqnas, kerZod, cnobili $\varphi(x)$ zemoqmedebiT (aRznebiT). naTqvams aqvs adgili, magaliTad droSi harmoniuli eleqtromagnituri da akustikuri talRebis difraqciis amocanebisaTvis sxvadasxva obieqtebze. am SemTxvevaSi $\varphi(x)$ aRzneba dacemul talRas warmoadgens. amocana ixsneba Semdegnairad: daSvebulia rom $g(y)$ ganawileba cnobilia da iwereba ucnobi gabneuli $f(x)$ velis zogadi gamosaxuleba

$$f(x) = \int_{(\Gamma)} G(x,y)\, g(y)\, dy.$$

aq (Γ) gambnevi obieqtis zedapiria. iwereba aseve sasazRvro piroba romelsac unda akmayofilebdes saZiebeli veli dacemul velTan erTad (Γ) zedapirze:

$$\hat{W}\left(f(x)+\varphi(x)\right)\Big|_{x\in(\Gamma)}=0.$$

aq $\hat{W}$ sasazRvro pirobis operatoria. Tu CavsvavT $f(x)$ funqciis gamosaxulebas am sasazRvro pirobaSi, maSin ucnobi $g(y)$ ganawilebis mimarT miviRebT integralur gantolebas

$$\int_{(\Gamma)}\hat{W}G(x,y)\Big|_{x\in(\Gamma)}g(y)dy=-\hat{W}\varphi(x)\Big|_{x\in(\Gamma)}.$$

grinis funqcias (Γ) zedapize singularoba gaaCnia, ris gamoc es integraluri gantoleba aseve singularulia.

imisaTvis rom Tavi avaridoT am singularobas, momentebis meTodi gvTavazobs moyvanili gantolebis marcxena mxareSi arsebuli integralis Canacvlebas wyaroebis diskretuli raodenobis jamiT, Tumca am SemTxvevaSi, sasazRvro piroba zedapiris yvela wertilSi ar sruldeba, rac iwvevs amonaxsnSi did cdomilebas.

v. kupraZis mier SemoTavazebul iqna singularobebis acilebis alternatiuli meTodi [31]. am meTodis saSualebiT sruliad ixsneba singularobis arsebobis problema (Γ) zedapirze, ris gamoc misi gamoyeneba bevrad ufro efeqturia. zogadi maTematikuri interpretaciidan gamomdinare [32], dgm mdgomareobs SemdegSi: kvlav ganvixiloT $f(x)$ funqciis gamosaxuleba da warmovidginoT igi Semdegi miaxloebuli mwkrivis saxiT

$$f(x)=\int_{(\Gamma)}G(x,y)g(y)dy\approx\sum_{n=1}^{N}g(y_n)dy_nG(x,y_n)=\sum_{n=1}^{N}a_nG(x,y_n),$$

sadac SemoRebulia aRniSvna $a_n=g(y_n)dy_n$ es mwkrivi warmoadgens sawyisi $f(x)$ funqciis gaSlas grinis funqciebiT sxvadasxva y_n argumentebisTvis da N s gazrdiT misi sizuste izrdeba. $G(x,y_n)$ funqciebis erToblioba, rodesac $n=1,2,\ldots$, qmnis srul wrfivad damoukidebel sistemas lebegis L^2 sivrceSi. v. kupraZis mier damtkicebul iqna, rom $f(x)$ funqcia, rogorc gabneuli veli, analizuria da SeiZleba misi analizuri gagrZeleba (Γ) zedapiridan. es iZleva saSualebas wavanacvloT y_n wertilebis zedapiri (Γ) zedapiridan garkveuli d manZiliT. am wanacvlebul y_n+d zedapirs damxmare zedapiri ewodeba, xolo masze ganlagebul wertilovan $a_nG(x,y_n)$ wyaroebs - damxmare gamomsxiveblebi. maSasadame moyvanili mwkrivis magivrad SegviZlia ganvixiloT mwkrivi

$$f(x)=\sum_{n=1}^{N}a_nG(x,y_n+d).$$

gaSlis ucnobi a_n koeficientebis gansazRvra, kupraZis Tanaxmad, SeiZleba moyvanili jamis wevrebis orTogonalizaciis saSualebiT sasazRvro pirobebis gamoyenebiT, romelic garkveul sirTuleebTanaa dakavSirebuli.

moyvanili gamosaxulebis Tanaxmad, saZiebeli $f(x)$ funqcia warmodgenilia cnobili grinis funqciebis jamis saxiT. yovel grinis funqciaSi figurirebs regularizaciis d parametric, romlis arsebobac uzrunvelyofs maT karg sigluves da Sesabamisad TviT $f(x)$ funqciis sigluves sxeulis $(\varGamma)$ zedapirze. aqedan gamomdinare moyvanil mwkrivs kargi krebadoba gaaCnia $(\varGamma)$ zedapiris gaswvriv da amitom ucnobi a_n koeficientebis sapovnelad ufro moxerxebulia kolokaciis meTodis gamoyeneba. amisaTvis CavsvaT aRniSnuli mwkrivis gamosaxuleba sasazRvro pirobaSi. miviRebT:

$$\sum_{n=1}^{N} a_n \hat{W} G\left(x, y_n + d\right)\Big|_{x\in(\varGamma)} = -\hat{W}\varphi\left(x\right)\Big|_{x\in(\varGamma)} .$$

Tu moviTxovT am sasazRvro pirobis Sesrulebas $(\varGamma)$ zedapiris N raodenobis sxvadasxva wertilSi (kolokaciis wertilebSi), maSin ucnobi a_n koeficientebis mimarT miviRebT wrfiv algebrul gantolebaTa sistemas

$$\sum_{n=1}^{N} a_n \hat{W} G\left(y_m, y_n + d\right) = -\hat{W}\varphi\left(y_m\right), \quad m = 1, 2, ..., N .$$

am gantolebaTa sistemas ar gaaCnia singularoba radgan grinis funqciis argumenti gansxvavdeba nulisgan maSinac ki, rodesac $m = n$. misi amoxsna Semdeg kompiuteruli modelirebis saSualebiT xdeba.

difraqciis amocanebSi, rogorc wesi, ganixileba droSi harmoniulad cvladi velebi:

$$f\left(x, t\right) = f\left(x\right) e^{-i\omega t} ,$$

rodesac drois maxasiaTebeli cnobilia da aris $e^{-i\omega t}$. sawyis operatorul gantolebas am SemTxvevaSi gaaCnia saxe

$$\Delta f\left(x\right) + k^2 f\left(x\right) = g\left(x\right)$$

da warmoadgens dalamberis talRur gantolebas kompleqsur formaSi. mas aseve araerTgvarovani helmhgolcis gantoleba ewodeba. Tu ganixileba organzomilebiani amocana, maSin grinis funqciis rols asrulebs hankelis funqcia:

$$G\left(x, y_n\right) = H_0^{(2)}\left(k\sqrt{\left(x - x_n\right)^2 + \left(y - y_n\right)^2}\right).$$

bolo ramodenime wlis ganmavlobaSi, Tsu-s gamoyenebiTi eleqtrodinamikis laboratoriis mkvlevarTa jgufi muSaobda damxmare gamomsxivneblebis meTodis gaumjobesebaze, raTa SesaZlebeli yofiliyo aRniSnuli meTodiT aseTi amocanebis amoxsna [33, 34]. sxva meTodebisgan gansxvavebiT, magaliTad momentebis meTodisgan gansxvavebiT [35], damxmare gamomsxivneblebis meTodis gamoyeneba mkveTrad amcirebs saWiro ucnobebis ricxvs, iZleva maRal sizustes da swraf krebadobas. Cvens jgufSi ganviTarebuli dgm gaxda mZlavri iaraRi zemoT aRniSnuli problemebis Sesaswavlad.

miRebuli Sedegebis samarTlianoba. sadisertacio naSromis farglebSi miRebuli ricxviTi eqsperimentebis Sedegebis marTebuloba mowmdeba sxeulis zedapirze sasazRvro pirobebis SesrulebiT da aseve maTi fizikuri arsis gaanalizebiT. agreTve, kerZo SemTxvevebSi, analizurad miRebul SedegebTan SedarebiT.

sadisertacio naSromis mokle mimoxilva. sadisertacio naSromi Sedgeba sami Tavisgan. yoveli Tavi iwyeba amocanis dasmiT da Sesabamisi maTematikuri aparatis CamoyalibebiT romlis saSualebiT ixsneba es amocana. Semdeg moyvanilia, Sesabamisi ricxviTi eqsperimentebis Sedegebi, romelnic miRebulia kompiuteruli modelirebis saSualebiT. yovel amocanisaTvis Seqmnilia programuli paketi, romelSic SesaZloa struqturis yvela parametris Secvla da mravali saintereso ricxviTi eqsperimentebis Catareba maRali sizustiT.

disertaciis pirvel TavSi ganixileba droSi harmoniuli eleqtromagnituri talRis difraqcia, sasruli zomebis meserze, romelic moTavsebulia Tavisufal sivrceSi da Semdeg sasruli zomebis mqone dieleqtrikSi. miRebulia kiraluri da aseve uaryofiTi gardatexis mqone struqturebi. difraqciis amocanasTan erTad ganixileba antenuri amocana, rodesac dacemuli velis wyaro struqturis SigniT imyofeba. kerZod, napovnia struqturis parametrebi brtyeli diagramis, kiraluri Tvisebebis da aseve uaryofiTi gardatexis misaRebad.

meore TavSi ganixileba btryeli, droSi harmoniuli talRis difraqcia usasrulo orperiodul meserze. orperiodulS vuwodebT organzomilebian mesers, romelsac gaaCnia ori periodi urTierTmarTobuli mimarTulebiT. ganixileba mesris elementis formis ramodenime SemTxveva. miRebulia struqturis parametrebis mniSvnelobebi rodesac igi garkveul sixSireebisaTvis sruliad amrekl zedapirs Seesabameba, an aris sruliad gamWvirvale.

mesame Tavi exeba brtyeli, droSi harmoniuli talRis difraqcias sistemaze, romelsac qmnis usasrulo orperioduli meseri da brtyeli usasrulo dieleqtrikuli fena. ganixileba ori gansxvavebuli SemTxveva, rodesac meseri imyofeba fenis SigniT da aseve mis maxloblad. moyvanilia sami maTematikuri meTodi aseTi saxis amocanebis amosaxsnelad. moyvanilia aseve zogierTi miRebuli ricxviTi Sedegebi.

Tavi I

eleqtromagnituri talRis difraqcia Tavisufal sivrceSi da dieleqtrikSi moTavsebul sasruli zomebis samganzomilebian periodul meserze

zogadi mimoxilva

pirveli Tavi Sedgeba sam paragrafisgan:

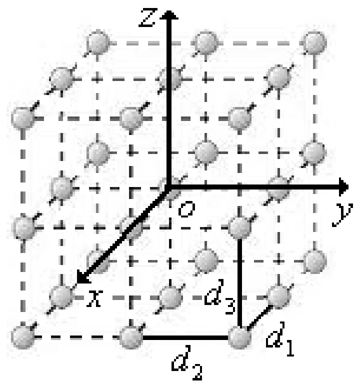
pirvel paragrafSi ganixileba difraqcia meserze, rodesac is Tavisufal garemoSi imyofeba. mesris elementebSi aRZruli denis ganawileba iZebneba Sesabamis sasazRvro pirobidan.

meore paragrafSi meseri moTavsebulia sasruli zomebis dieleqtrikSi. dieleqtriki SemosazRvrulia gluvi zedapiriT da gaaCnia kompleqsuri dieleqtrikuli SeRwevadoba. Sesabamisad mas gaaCnia danakargebi. aseTi metalo - dieleqtrikuli struqturis Teoriuli gamokvleva moiTxovs damxmare gamomsxiveblebis meTodebis gamoyenebas.

mesame paragrafSi moyvanilia Seqmnili kodis saSualebiT miRebuli ricxviTi eqsperimentis Sedegebi. ganixileba mesris oTxi sxvadasxva rezonansuli elementi. esenia: koncentrirebuli ori gamtari Ria rgoli, gamtari Ω elementi, gamtari spirali da aseve gamtari kasinis elementi. gamokvleuli struqturebis parametrebi mocemulia dayvanil erTeulebSi, rac xdis miRebul Sedegebs samarTlians sixSiris farTo diapazonSi, sadac SeiZleba gamoyenebul iqnas maqsvelis klasikuri Teoria.

$1.1 eleqtromagnituri talRis difraqcia sasruli zomebis meserze Tavisufal garemoSi

amocanis dasma. ganvixiloT Tavisufal garemoSi moTavsebuli sasruli zomebis samganzomilebiani meseri, romelic Sedgeba rezonansuli Tvisebebis mqone gamtar elementebisagan (nax. 1.1.1). mesris periodi sakoordinato RerZebis gaswvriv Sesabamisad avRniSnoT rogorc d_1, d_2, d_3. moxerxebulia ganxilul iqnas kerZo SemTxveva, rodesac yoveli sakoordinato RerZis gaswvriv gagvaCnia elementebis kenti raodenoba. amitom CavTvaloT, rom elementebis sruli raodenoba meserSi udris $(2N+1)(2M+1)(2P+1)$, sadac N, M, P fiqsirebuli ricxvebia.



nax. 1.1.1 perioduli meseri

mesris elementi warmoadgens mcire dr_0 radiusis mqone gamtars. unda aRiniSnos, rom Camoyalibebuli Teoria aris zogadi da amitom araa dakonkretebuli Tu ra forma gaaCnia am elements. es Teoria SeiZleba gamoyenebul iqnas sxvadasxva elementebis SemTxvevaSi. mesris periodebi unda aRematebodes misi elementebis zomebs, imisaTvis rom ar hqondes adgili elementebis urTierTgadakveTas.

ganxilul mesers ecema cnobili, droSi harmoniulad cvladi, eleqtromagnituri talRa $\vec{E}_{inc}(\vec{r})$, $\vec{H}_{inc}(\vec{r})$, sadac $\vec{r}$ dakvirvebis wertilis radiusveqtoria. drois maxasiaTebelia $e^{-i\omega t}$. dacemuli talRa aCens mesris yovel elementSi denis da muxtis ganawilebas, rac warmoadgens meoradi (gabneuli velis) wyaros. Cveni amocanaa vipovoT denis da muxtis es ganawileba da Sesabamisad difraqciis Sedegad mesridan gabneuli $\vec{E}_s(\vec{r})$, $\vec{H}_s(\vec{r})$ veli.

amocanis amoxsnis meTodi. mesris centraluri elementis gaswvriv ganvixiloT $\vec{r}_0\{x_0, y_0, z_0\}$ radiusveqtori. maSin sxva elementis radiusveqtori, romlis nomeria n,m,p, Caiwereba rogorc $\vec{r}_{n,m,p}\{x_n, y_m, z_p\}$, sadac

$$\begin{cases} x_n = x_0 + nd_1, \\ y_m = y_0 + md_2, \\ z_p = z_0 + pd_3, \end{cases} \qquad \begin{cases} -N \le n \le N, \\ -M \le m \le M, \\ -P \le p \le P. \end{cases}$$

ucnobi gabneuli veli unda akmayofilebdes maqsvelis gantolebaTa sistemas da sasazRvro pirobas mesris yoveli elementis zedapiris gaswvriv:

$$\vec{E}_{inc}\left(\vec{r}_{n',m',p'} + d\vec{r}_0\right)\cdot\vec{\tau}_0 + \vec{E}_s\left(\vec{r}_{n',m',p'} + d\vec{r}_0\right)\cdot\vec{\tau}_0 = 0\,, \qquad \begin{cases} -N \le n' \le N, \\ -M \le m' \le M, \\ -P \le p' \le P, \end{cases} \qquad (1.1.1)$$

sadac $\vec{\tau}_0$ erTeulovani tangencialuri veqtoria.

gabneuli veli $\vec{r}$ dakvirvebis wertilSi Sedgeba yoveli elementidan wamosul velebisgan:

$$\vec{E}_s(\vec{r}) = \sum_{n,m,p} \vec{E}_{n,m,p}(\vec{r}), \qquad \vec{H}_s(\vec{r}) = \sum_{n,m,p} \vec{H}_{n,m,p}(\vec{r}).$$

am velebis sapovnelad moxerxebulia gamoyenebul iqnas potencialebis meTodi, romlis Tanaxmad ucnobi gabneuli veli SegviZlia warmovidginoT rogorc

$$\vec{E}_s(\vec{r}) = -grad\varphi(\vec{r}) + i\omega\vec{A}(\vec{r}), \qquad \vec{H}_s(\vec{r}) = (1/\mu_0)\,rot\vec{A}(\vec{r}),$$

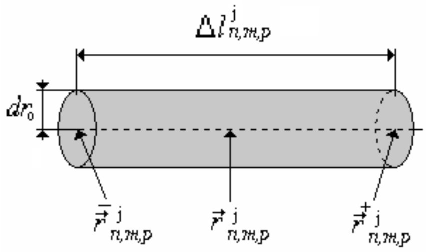
sadac

$$\vec{A}(\vec{r}) = (\mu_0/4\pi)\sum_{n,m,p}\int_{l_{n,m,p}} I_{n,m,p} G\left(\vec{r},\vec{r}_{n,m,p}\right) d\vec{l}\,, \quad \varphi(\vec{r}) = (1/4\pi\varepsilon_0)\sum_{n,m,p}\int_{l_{n,m,p}} \sigma_{n,m,p} G\left(\vec{r},\vec{r}_{n,m,p}\right) dl\,, \qquad (1.1.2)$$

$$G\left(\vec{r},\vec{r}_{n,m,p}\right) = e^{ik_0\left|\vec{r}-\vec{r}_{n,m,p}\right|} \Big/ \left|\vec{r}-\vec{r}_{n,m,p}\right|, \quad k_0 = \omega\sqrt{\mu_0\varepsilon_0}\,, \quad \sigma_{n,m,p} = -(i/\omega)\,dI_{n,m,p}/dl\,. \qquad (1.1.3)$$

aq $I_{n,m,p}$ da $\sigma_{n,m,p}$ mesris $l_{n,m,p}$ elementSi aRZruli ucnobi denis da muxtebis ganawilebaa.

davyoT mesris yoveli elementi didi K raodenobis mcire sigrZis $\Delta l^{j}_{n,m,p}$ segmentebad ($j=1,2,...,K$). yoveli segmenti davaxasiaToT centraluri $\vec{r}^{\,j}_{n,m,p}$ da $\overset{+}{\vec{r}}{}^{j}_{n,m,p}$, $\overset{-}{\vec{r}}{}^{j}_{n,m,p}$ kidura wertilebiT (nax. 1.1.2).



Nnax. 1.1.2 segmentis geometria mesris elementSi

segmentebis K raodenoba aviRoT sakmarisi imisaTvis rom SegveZlos ugulebelvyoT denis $I_{n,m,p}$ cvlileba yovel maTganis gaswvriv. maSasadame CavTvaloT, rom yovel $\Delta l^{j}_{n,m,p}$ segmentSi gaedineba mudmivi amplitudis $I^{j}_{n,m,p}$ deni. es SesaZloa im SemTxvevaSi, Tu am segmentis boloebSi imyofeba ori urTierTsapirispiro niSnis muxti $+q^{j}_{n,m,p}$ da $-q^{j}_{n,m,p}$, sadac uwyvetobis gantolebidan gamomdinare

$$q^{j}_{n,m,p}=-(i/\omega)I^{j}_{n,m,p}.$$

naTqvamis gaTvaliswinebiT potencialebis gamosaxulebebSi integrireba mesris elementis gaswvriv unda CavanacvloT jamiT:

integralisTvis, romelic veqtoruli potencialis gamosaxulebaSi figurirebs, miviRebT:

$$\int\limits_{l_{n,m,p}} I_{n,m,p}G\left(\vec{r},\vec{r}_{n,m,p}\right)d\vec{l}\approx\sum_{j}I^{j}_{n,m,p}G\left(\vec{r},\vec{r}^{\,j}_{n,m,p}\right)\Delta\vec{l}^{\,j}_{n,m,p}.$$

skalaruli potencialisTvis mivaqcioT yuradReba imas rom TiToeul $\Delta l^{j}_{n,m,p}$ segmentze imyofeba ori muxti. amis gamo unda davweroT

$$\int\limits_{l_{n,m,p}} \sigma_{n,m,p}G\left(\vec{r},\vec{r}_{n,m,p}\right)dl\approx-(i/\omega)\sum_{j}I^{j}_{n,m,p}\left[G\left(\vec{r},\overset{+}{\vec{r}}{}^{j}_{n,m,p}\right)-G\left(\vec{r},\overset{-}{\vec{r}}{}^{j}_{n,m,p}\right)\right]=$$

$$=-(i/\omega)\sum_{j}I^{j}_{n,m,p}\Delta G\left(\vec{r},\vec{r}^{\,j}_{n,m,p}\right).$$

unda aRiniSnos, rom sasruli sxvaoba $\Delta G\left(\vec{r},\vec{r}^{\,j}_{n,m,p}\right)$ bolo gamosaxulebaSi ar icvleba diferencialiT, radgan aseTi Canacvleba gaxdeba samarTliani elementaruli segmentebis ufro did raodenobasaTvis, rac modelirebisas moiTxovs kompiuteris gacilebiT ufro met resursebs.

Tu SevitanT integralebis miRebul mniSvnelobebs potencialebis (1.1.2) gamosaxulebebSi, maSin gveqneba:

$$\vec{A}(\vec{r})=(\mu_0/4\pi)\sum_{n,m,p}\sum_{j}I^{j}_{n,m,p}G\left(\vec{r},\vec{r}^{\,j}_{n,m,p}\right)\Delta\vec{l}^{\,j}_{n,m,p},\qquad \varphi(\vec{r})=-(i/4\pi\omega\varepsilon_0)\sum_{n,m,p}\sum_{j}I^{j}_{n,m,p}\Delta G\left(\vec{r},\vec{r}^{\,j}_{n,m,p}\right),$$

$$\Delta G\left(\vec{r},\vec{r}^{\,j}_{n,m,p}\right)=G\left(\vec{r},\overset{+}{\vec{r}}{}^{j}_{n,m,p}\right)-G\left(\vec{r},\overset{-}{\vec{r}}{}^{j}_{n,m,p}\right).$$

gabneuli velis gamosaxuleba. veqtoruli da skalaruli potencialebis meSveobiT SegviZlia gamovsaxoT ucnobi gabneuli veli:

$$\vec{E}_s(\vec{r}) = -grad\varphi(\vec{r}) + i\omega\vec{A}(\vec{r}), \qquad \vec{H}_s(\vec{r}) = (1/\mu_0)\,rot\vec{A}(\vec{r}).$$

Tu SemovitanT aRniSvnas

$$\tilde{G}(\vec{r},\vec{r}^{\,j}_{n,m,p}) = e^{ik_0|\vec{r}-\vec{r}^{\,j}_{n,m,p}|}\left(ik_0|\vec{r}-\vec{r}^{\,j}_{n,m,p}|-1\right)\Big/\left|\vec{r}-\vec{r}^{\,j}_{n,m,p}\right|^3, \tag{1.1.4}$$

maSin garkveuli gamoTvlebis Sedegad miviRebT

$$rot\vec{A}(\vec{r}) = (\mu_0/4\pi)\sum_{n,m,p}\sum_j I^j_{n,m,p}\tilde{G}(\vec{r},\vec{r}^{\,j}_{n,m,p})(\vec{r}-\vec{r}^{\,j}_{n,m,p})\times\Delta\vec{l}^{\,j}_{n,m,p},$$

$$grad\varphi(\vec{r}) = -(i/4\pi\omega\varepsilon_0)\sum_{n,m,p}\sum_j I^j_{n,m,p}\Delta\left[\tilde{G}(\vec{r},\vec{r}^{\,j}_{n,m,p})(\vec{r}-\vec{r}^{\,j}_{n,m,p})\right],$$

sadac

$$\Delta\left[\tilde{G}(\vec{r},\vec{r}^{\,j}_{n,m,p})(\vec{r}-\vec{r}^{\,j}_{n,m,p})\right] = \tilde{G}\left(\vec{r},\overset{+}{\vec{r}}{}^{\,j}_{n,m,p}\right)\left(\vec{r}-\overset{+}{\vec{r}}{}^{\,j}_{n,m,p}\right) - \tilde{G}\left(\vec{r},\overset{-}{\vec{r}}{}^{\,j}_{n,m,p}\right)\left(\vec{r}-\overset{-}{\vec{r}}{}^{\,j}_{n,m,p}\right).$$

gabneuli veli Semdegnairad gamoisaxeba:

$$\vec{E}_s(\vec{r}) = (i/4\pi\omega\varepsilon_0)\sum_{n,m,p}\sum_j I^j_{n,m,p}\left\{k_0^2 G(\vec{r},\vec{r}^{\,j}_{n,m,p})\Delta\vec{l}^{\,j}_{n,m,p} + \Delta\left[\tilde{G}(\vec{r},\vec{r}^{\,j}_{n,m,p})(\vec{r}-\vec{r}^{\,j}_{n,m,p})\right]\right\}, \tag{1.1.5}$$

$$\vec{H}_s(\vec{r}) = (1/4\pi)\sum_{n,m,p}\sum_j I^j_{n,m,p}\tilde{G}(\vec{r},\vec{r}^{\,j}_{n,m,p})(\vec{r}-\vec{r}^{\,j}_{n,m,p})\times\Delta\vec{l}^{\,j}_{n,m,p}. \tag{1.1.6}$$

denis ganawilebis gansazRvra mesris elementebSi. denebis ucnobi $I^j_{n,m,p}$ amplitudebs vipoviT sasazRvro pirobidan, romelsac gabneuli veli unda akmayofilebdes yovel $\Delta l^k_{n',m',p'}$ segmentis gaswvriv. sasazRvro pirobidan gamomdinare

$$\vec{E}_s\left(\vec{r}^{\,k}_{n',m',p'} + d\vec{r}_0\right)\cdot\Delta\vec{l}^{\,k}_{n',m',p'} = -\vec{E}_{inc}\left(\vec{r}^{\,k}_{n',m',p'} + d\vec{r}_0\right)\cdot\Delta\vec{l}^{\,k}_{n',m',p'}.$$

Tu movaxdenT velis gamosaxulebis Casmas am pirobaSi, maSin ucnobi amplitudebis mimarT miviRebT wrfiv algebrul gantolebaTa sistemas:

$$\sum_{n,m,p}\sum_j Z^{j,k}_{n,m,p,n',m',p'} I^j_{n,m,p} = \vec{E}_{inc}\left(\vec{r}^{\,k}_{n',m',p'} + d\vec{r}_0\right)\cdot\Delta\vec{l}^{\,k}_{n',m',p'}, \tag{1.1.7}$$

$$\begin{cases} -N \le n' \le N, \\ -M \le m' \le M, \qquad k = 1,2,\ldots, \\ -P \le p' \le P, \end{cases}$$

sadac

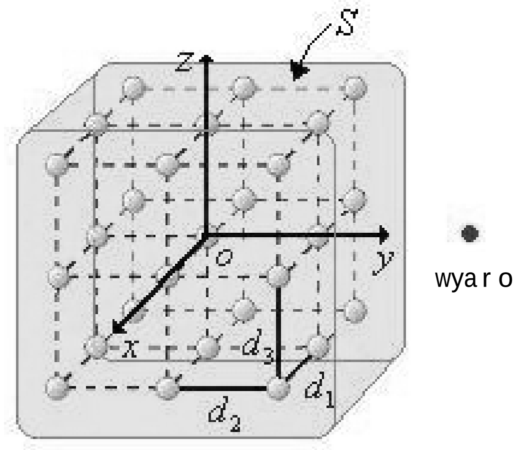
$$Z^{j,k}_{n,m,p,n',m',p'} = -(i/4\pi\omega\varepsilon_0)\times$$

$$\times\left\{k_0^2 G\left(\vec{r}^{\,k}_{n',m',p'} + d\vec{r}_0, \vec{r}^{\,j}_{n,m,p}\right)\Delta\vec{l}^{\,j}_{n,m,p} + \Delta\left[\tilde{G}\left(\vec{r}^{\,k}_{n',m',p'} + d\vec{r}_0, \vec{r}^{\,j}_{n,m,p}\right)\left(\vec{r}^{\,k}_{n',m',p'} - \vec{r}^{\,j}_{n,m,p} + d\vec{r}_0\right)\right]\right\}\cdot\Delta\vec{l}^{\,k}_{n',m',p'}. \tag{1.1.8}$$

gantolebebis raodenoba sistemaSi tolia $K(2N+1)(2M+1)(2P+1)$, rac udris ucnobi amplitudebis raodenobas. am sistemis amoxsna kompiuteris saSualebiT xdeba, ris Semdegadac ucnobi gabneuli veli (1.1.5) da (1.1.6) formulebis saxiT Caiwereba.

$1.2 difraqcia dieleqtrikSi moTavsebul periodul meserze

amocanis dasma. davuSvaT rom, zemoT ganxiluli meseri moTavsebulia sasruli zomebis $\varepsilon=\varepsilon'+i\varepsilon''$ kompleqsuri dieleqtrikuli da μ magnituri SeRwevadobebis mqone dieleqtrikSi (nax. 1.2.1). dieleqtriki SemosazRvrulia gluvi S zedapiriT. sivrcis garkveul wertilSi imyofeba wertilovani, cnobili eleqtromagnituri $\vec{E}_{inc}(\vec{r})$, $\vec{H}_{inc}(\vec{r})$ talRis wyaro. amocanaa vipovoT difraqciis Sedegad miRebuli velis struqturis SigniT da gareT. Cven unda ganvixiloT ori gansxvavebuli SemTxveva, rodesac dacemuli velis wyaro imyofeba struqturis gareT da mis SigniT. pirvel SemTxvevaSi, Tu igi Sor manZilze imyofeba struqturidan, maSin gveqneba brtyeli dacemuli $\vec{E}_{inc}(\vec{r})$, $\vec{H}_{inc}(\vec{r})$ talRa.



nax. 1.2.1 dieleqtrikSi Casmuli perioduli meseri

Tu dacemuli velis wyaro struqturis gareT imyofeba, maSin gare sruli $\vec{E}_{out}(\vec{r})$, $\vec{H}_{out}(\vec{r})$ veli Sedgeba dacemuli $\vec{E}_{inc}(\vec{r})$, $\vec{H}_{inc}(\vec{r})$ da dieleqtrikis S zedapiridan gabneuli ucnobi $\vec{E}_1(\vec{r})$, $\vec{H}_1(\vec{r})$ velebisagan:

$$\vec{E}_{out}(\vec{r})=\vec{E}_1(\vec{r})+\vec{E}_{inc}(\vec{r}),\quad \vec{H}_{out}(\vec{r})=\vec{H}_1(\vec{r})+\vec{H}_{inc}(\vec{r}).$$

struqturis SigniT sruli $\vec{E}_{in}(\vec{r})$, $\vec{H}_{in}(\vec{r})$ veli Sedgeba mesris mier gabneul $\vec{E}_s(\vec{r})$, $\vec{H}_s(\vec{r})$ da dieleqtrikis S zedapiridan SigniT gabneul ucnob $\vec{E}_2(\vec{r})$, $\vec{H}_2(\vec{r})$ velebisagan:

$$\vec{E}_{in}(\vec{r})=\vec{E}_2(\vec{r})+\vec{E}_s(\vec{r}),\quad \vec{H}_{in}(\vec{r})=\vec{H}_2(\vec{r})+\vec{H}_s(\vec{r}).$$

im SemTxvevaSi, rodesac dacemuli velis wyaro struqturis SigniT imyofeba, sruli gare veli warmoadgens mxolod im ucnob $\vec{E}_1(\vec{r})$, $\vec{H}_1(\vec{r})$ vels, romelsac dieleqtrikis S zedapiri asxivebs:

$$\vec{E}_{out}(\vec{r})=\vec{E}_1(\vec{r}),\quad \vec{H}_{out}(\vec{r})=\vec{H}_1(\vec{r}).$$

sruli Sida veli am SemTxvevaSi Sedgeba dacemuli $\vec{E}_{inc}(\vec{r})$, $\vec{H}_{inc}(\vec{r})$, mesridan gabneuli $\vec{E}_s(\vec{r})$, $\vec{H}_s(\vec{r})$ da S zedapiridan SigniT gadasxivebuli $\vec{E}_2(\vec{r})$, $\vec{H}_2(\vec{r})$ velebisagan:

$$\vec{E}_{in}(\vec{r})=\vec{E}_2(\vec{r})+\vec{E}_s(\vec{r})+\vec{E}_{inc}(\vec{r}),\ \ \vec{H}_{in}(\vec{r})=\vec{H}_2(\vec{r})+\vec{H}_s(\vec{r})+\vec{H}_{inc}(\vec{r}).$$

sruli Sida da gare velebi unda akmayofilebdnen sasazRvro pirobebs dieleqtrikis S zedapirze da aseve mesris yoveli elenmentis gaswvriv.

dieleqtrikis S zedapirze moiTxoveba sruli velis tangencialuri mdgenelis uwyvetobis piroba. Cven ganvixilavT difraqciis amocanas samganzomilebian struqturaze da maSasadame, sasazRvro piroba unda sruldebodes S zedapiris nebismieri tangencialis gaswvriv. avagoT am zedapiris yovel wertilSi ori urTierTmarTobuli erTeulovani mxebi veqtori $\vec{\tau}_1$ da $\vec{\tau}_2$. gasagebia rom Tu sasazRvro piroba sruldeba orive veqtoris gaswvriv maSin is nebismieri sxva tangencialis gaswvriv Sesruldeba. naTqvamis Tanaxmad

$$\begin{cases}\vec{E}_{in}(\vec{r}_S)\cdot\tau_1=\vec{E}_{out}(\vec{r}_S)\cdot\tau_1,\\ \vec{H}_{in}(\vec{r}_S)\cdot\tau_1=\vec{H}_{out}(\vec{r}_S)\cdot\tau_1,\\ \vec{E}_{in}(\vec{r}_S)\cdot\tau_2=\vec{E}_{out}(\vec{r}_S)\cdot\tau_2,\\ \vec{H}_{in}(\vec{r}_S)\cdot\tau_2=\vec{H}_{out}(\vec{r}_S)\cdot\tau_2,\end{cases}\qquad(1.2.1)$$

sadac $\vec{r}_S$ dieleqtrikis S zedapiris wertilis radiusveqtoria.

mesris elementebisaTvis moiTxoveba sruli Sida velis tangencialuri mdgenelis nulTan tolobis piroba:

$$\vec{E}_{in}\left(\vec{r}_{n',m',p'}+d\vec{r}_0\right)\cdot\vec{\tau}_0=0.\qquad(1.2.2)$$

pirvel SemTxvevaSi, rodesac dacemuli velis wyaro gareT imyofeba, (1.2.1) da (1.2.2) sasazRvro pirobebidan miviRebT:

$$\begin{cases}\vec{E}_2(\vec{r}_S)\cdot\tau_1-\vec{E}_1(\vec{r}_S)\cdot\tau_1+\vec{E}_s(\vec{r}_S)\cdot\tau_1=\vec{E}_{inc}(\vec{r}_S)\cdot\tau_1,\\ \vec{H}_2(\vec{r}_S)\cdot\tau_1-\vec{H}_1(\vec{r}_S)\cdot\tau_1+\vec{H}_s(\vec{r}_S)\cdot\tau_1=\vec{H}_{inc}(\vec{r}_S)\cdot\tau_1,\\ \vec{E}_2(\vec{r}_S)\cdot\tau_2-\vec{E}_1(\vec{r}_S)\cdot\tau_2+\vec{E}_s(\vec{r}_S)\cdot\tau_2=\vec{E}_{inc}(\vec{r}_S)\cdot\tau_2,\\ \vec{H}_2(\vec{r}_S)\cdot\tau_2-\vec{H}_1(\vec{r}_S)\cdot\tau_2+\vec{H}_s(\vec{r}_S)\cdot\tau_2=\vec{H}_{inc}(\vec{r}_S)\cdot\tau_2,\end{cases}$$

$$\vec{E}_s\left(\vec{r}_{n',m',p'}+d\vec{r}\right)\cdot\vec{\tau}_0+\vec{E}_2\left(\vec{r}_{n',m',p'}+d\vec{r}\right)\cdot\vec{\tau}_0=0.$$

meore SemTxvevaSi, analogiurad gveqneba

$$\begin{cases}\vec{E}_2(\vec{r}_S)\cdot\tau_1-\vec{E}_1(\vec{r}_S)\cdot\tau_1+\vec{E}_s(\vec{r}_S)\cdot\tau_1=-\vec{E}_{inc}(\vec{r}_S)\cdot\tau_1,\\ \vec{H}_2(\vec{r}_S)\cdot\tau_1-\vec{H}_1(\vec{r}_S)\cdot\tau_1+\vec{H}_s(\vec{r}_S)\cdot\tau_1=-\vec{H}_{inc}(\vec{r}_S)\cdot\tau_1,\\ \vec{E}_2(\vec{r}_S)\cdot\tau_2-\vec{E}_1(\vec{r}_S)\cdot\tau_2+\vec{E}_s(\vec{r}_S)\cdot\tau_2=-\vec{E}_{inc}(\vec{r}_S)\cdot\tau_2,\\ \vec{H}_2(\vec{r}_S)\cdot\tau_2-\vec{H}_1(\vec{r}_S)\cdot\tau_2+\vec{H}_s(\vec{r}_S)\cdot\tau_2=-\vec{H}_{inc}(\vec{r}_S)\cdot\tau_2,\end{cases}$$

$$\vec{E}_s\left(\vec{r}_{n',m',p'}+d\vec{r}_0\right)\cdot\vec{\tau}_0+\vec{E}_2\left(\vec{r}_{n',m',p'}+d\vec{r}_0\right)\cdot\vec{\tau}_0=-\vec{E}_{inc}\left(\vec{r}_{n',m',p'}+d\vec{r}_0\right)\cdot\vec{\tau}_0.$$

dasmuli amocanis Tanaxmad, unda vipovoT difraqciis Sedegad miRebuli velebi dieleqtrikis SigniT da gareT da aseve unda davakmayofiloT (1.2.1), (1.2.2) sasazRvro pirobebi.

amocanis amoxsnis meTodi. wina paragrafSi iqna napovni mesris mier gabneuli ucnobi $\vec{E}_s(\vec{r})$, $\vec{H}_s(\vec{r})$ veli. misi damokidebuleba ucnobi denis $I^j_{n,m,p}$ amplitudebisagan Semdegi formulebiT gamoisaxeba:

$$\vec{E}_s(\vec{r}) = (i/4\pi\omega\varepsilon_0\varepsilon)\sum_{n,m,p}\sum_j I^j_{n,m,p}\left\{k^2 G(\vec{r},\vec{r}^{\,j}_{n,m,p})\Delta\vec{l}^{\,j}_{n,m,p} + \Delta\left[\tilde{G}(\vec{r},\vec{r}^{\,j}_{n,m,p})(\vec{r}-\vec{r}^{\,j}_{n,m,p})\right]\right\}, \quad (1.2.3)$$

$$\vec{H}_s(\vec{r}) = (1/4\pi)\sum_{n,m,p}\sum_j I^j_{n,m,p}\tilde{G}(\vec{r},\vec{r}^{\,j}_{n,m,p})(\vec{r}-\vec{r}^{\,j}_{n,m,p})\times\Delta\vec{l}^{\,j}_{n,m,p}, \quad (1.2.4)$$

sadac

$$k = \omega\sqrt{\varepsilon_0\varepsilon\mu_0\mu}, \quad G(\vec{r},\vec{r}^{\,j}_{n,m,p}) = e^{ik|\vec{r}-\vec{r}^{\,j}_{n,m,p}|}\Big/|\vec{r}-\vec{r}^{\,j}_{n,m,p}|,$$

$$\tilde{G}(\vec{r},\vec{r}^{\,j}_{n,m,p}) = e^{ik|\vec{r}-\vec{r}^{\,i}_{n,m,p}|}\left(ik|\vec{r}-\vec{r}^{\,j}_{n,m,p}|-1\right)\Big/|\vec{r}-\vec{r}^{\,j}_{n,m,p}|^3,$$

$$\Delta\left[\tilde{G}(\vec{r},\vec{r}^{\,j}_{n,m,p})(\vec{r}-\vec{r}^{\,j}_{n,m,p})\right] = \tilde{G}\left(\vec{r},\overset{+}{\vec{r}}{}^{j}_{n,m,p}\right)\left(\vec{r}-\overset{+}{\vec{r}}{}^{j}_{n,m,p}\right) - \tilde{G}\left(\vec{r},\overset{-}{\vec{r}}{}^{j}_{n,m,p}\right)\left(\vec{r}-\overset{-}{\vec{r}}{}^{j}_{n,m,p}\right).$$

ucnobi $\vec{E}_1(\vec{r})$, $\vec{H}_1(\vec{r})$ da $\vec{E}_2(\vec{r})$, $\vec{H}_2(\vec{r})$ velebis sapovnelad gamoviyenoT damxmare gamomsxiveblebis meTodi. amisaTvis dieleqtrikis zedapiris orive mxridan, Tanabari daSorebiT avagoT ori gluvi damxmare zedapiri S_{in}, S_{out} da ganvalagoT yovel maTganze damxmare wyaroebi. damxmare wyaroebis raodenoba yovel aseT zedapirze unda udrides wertilebis im raodenobas dieleqtrikis S zedapirze, sadac Sesabamisi (1.2.1) sasazRvro pirobis Sesruleba moiTxoveba.

damxmare wyarod moxerxebulia arCeuli iqnas ori urTierTmarTobuli kombinirebuli dipoli. maTi saSualebiT SeiZleba warmodgenili iqnas nebismieri saxis da nebismieri polarizaciis mqone veli. am dipolebis polarizaciis veqtorebi, damxmare zedapiris yovel wertilSi mivmarToT $\vec{\tau}_1$ da $\vec{\tau}_2$ tangencialebis paralelurad. Sida wyarobis meSveobiT aRiwereba gare ucnobi $\vec{E}_1(\vec{r})$, $\vec{H}_1(\vec{r})$ veli. gare damxmare wyaroebi ki aRweren dieleqtrikis SigniT ucnob $\vec{E}_2(\vec{r})$, $\vec{H}_2(\vec{r})$ vels.

damxmare wyaros mier gamosxivebuli velisTvis moxerxebulia SemoviRoT aRniSvna $\vec{E}^{\alpha,\beta}_{in,out(\tau_i)}$. aq gvaqvs zeda da qveda indeqsebi. zeda indeqsebi gviCveneben damxmare wyaros nomers $(\alpha,\beta = 1,2,...)$. pirveli an meore qveda indeqsis mixedviT gavigebT romel damxmare zedapirs ekuTvnis es dipoli, frCxilebSi mdebare indeqsi gviCvenebs dipolis orientacias.

yovel kombinirebul dipoles, damxmare gamomsxivebelSi gaaCnia Tavisi ucnobi amplituda. amitom, Tu ganvixilavT (α, β) damxmare wyaros Sida S_{in} zedapirze, maSin misi veli iqneba

$$A_{\alpha,\beta}\vec{E}^{\alpha,\beta}_{in(\tau_1)}(\vec{r}) + B_{\alpha,\beta}\vec{E}^{\alpha,\beta}_{in(\tau_2)}(\vec{r}), \qquad A_{\alpha,\beta}\vec{H}^{\alpha,\beta}_{in(\tau_1)}(\vec{r}) + B_{\alpha,\beta}\vec{H}^{\alpha,\beta}_{in(\tau_2)}(\vec{r}).$$

analogiurad, gare S_{out} zedapiris (γ, δ) wyarosTvis gveqneba

$$C_{\gamma,\delta}\vec{E}^{\gamma,\delta}_{out(\tau_1)}(\vec{r}) + D_{\gamma,\delta}\vec{E}^{\gamma,\delta}_{out(\tau_2)}(\vec{r}), \qquad C_{\gamma,\delta}\vec{H}^{\gamma,\delta}_{out(\tau_1)}(\vec{r}) + D_{\gamma,\delta}\vec{H}^{\gamma,\delta}_{out(\tau_2)}(\vec{r}).$$

aq $A_{\alpha,\beta}$, $B_{\alpha,\beta}$, $C_{\gamma,\delta}$, $D_{\gamma,\delta}$ damxmare wyaroebis ucnobi amplitudebia.

kombinirebuli dipolis mier gamosxivebuli velis zogadi gamosaxuleba cnobilia Teoriidan [36]:

$$\vec{E}\left(\vec{R}\right)=\vec{E}_{el}\left(\vec{R}\right)+\sqrt{\mu_0\mu/\varepsilon_0\varepsilon}\,\vec{E}_{mag}\left(\vec{R}\right),\qquad \vec{H}\left(\vec{R}\right)=\vec{H}_{el}\left(\vec{R}\right)+\sqrt{\mu_0\mu/\varepsilon_0\varepsilon}\,\vec{H}_{mag}\left(\vec{R}\right),$$

sadac

$$\vec{E}_{el}\left(\vec{R}\right)=\left(e^{ikR}/4\pi\varepsilon_0\varepsilon\right)\left\{\left(1/R^3-ik/R^2\right)\left[3\vec{R}_0\cdot\left(\vec{R}_0\cdot\vec{\tau}_{el}\right)-\vec{\tau}_{el}\right]-\left(k^2/R\right)\left(\vec{R}_0\times\left(\vec{R}_0\times\vec{\tau}_{el}\right)\right)\right\},$$

$$\vec{H}_{el}\left(\vec{R}\right)=-\left(i\omega e^{ikR}/4\pi\right)\left(1/R^2-ik/R\right)\left(\vec{\tau}_{el}\times\vec{R}_0\right);$$

$$\vec{E}_{mag}\left(\vec{R}\right)=\left(k^2e^{ikR}/4\pi\right)\left(1/R^2-ik/R\right)\left(\vec{\tau}_{mag}\times\vec{R}_0\right);$$

$$\vec{H}_{mag}\left(\vec{R}\right)=\left(e^{ikR}/4\pi\right)\left\{\left(1/R^3-ik/R^2\right)\left[3\vec{R}_0\cdot\left(\vec{R}_0\cdot\vec{\tau}_{mag}\right)-\vec{\tau}_{mag}\right]-\left(k^2/R\right)\left(\vec{R}_0\times\left(\vec{R}_0\times\vec{\tau}_{mag}\right)\right)\right\}.$$

aq $\vec{R}_0$ erTeulovani veqtoria, romelic aris mimarTuli dipolidan dakvirvebis wertilSi, R warmoadgens manZils dipolsa da dakvirvebis wertils Soris, xolo $\vec{\tau}$ dipolis polarizaciis erTeulovani veqtoria

ucnobi $\vec{E}_1(\vec{r})$, $\vec{H}_1(\vec{r})$ da $\vec{E}_2(\vec{r})$, $\vec{H}_2(\vec{r})$ velebisTvis gveqneba:

$$\vec{E}_1(\vec{r})=\sum_{\alpha,\beta}\left[A_{\alpha,\beta}\vec{E}^{\alpha,\beta}_{in(\tau_1)}(\vec{r})+B_{\alpha,\beta}\vec{E}^{\alpha,\beta}_{in(\tau_2)}(\vec{r})\right], \qquad (1.2.5)$$

$$\vec{H}_1(\vec{r})=\sum_{\alpha,\beta}\left[A_{\alpha,\beta}\vec{H}^{\alpha,\beta}_{in(\tau_1)}(\vec{r})+B_{\alpha,\beta}\vec{H}^{\alpha,\beta}_{in(\tau_2)}(\vec{r})\right], \qquad (1.2.6)$$

$$\vec{E}_2(\vec{r})=\sum_{\gamma,\delta}\left[C_{\gamma,\delta}\vec{E}^{\gamma,\delta}_{out(\tau_1)}(\vec{r})+D_{\gamma,\delta}\vec{E}^{\gamma,\delta}_{out(\tau_2)}(\vec{r})\right], \qquad (1.2.7)$$

$$\vec{H}_2(\vec{r})=\sum_{\gamma,\delta}\left[C_{\gamma,\delta}\vec{H}^{\gamma,\delta}_{out(\tau_1)}(\vec{r})+D_{\gamma,\delta}\vec{H}^{\gamma,\delta}_{out(\tau_2)}(\vec{r})\right]. \qquad (1.2.8)$$

maSasadame amocana dayvanilia damxmare wyaroebis ucnobi $A_{\alpha,\beta}$, $B_{\alpha,\beta}$, $C_{\gamma,\delta}$, $D_{\gamma,\delta}$ amplitudebis da mesris elementebSi aRZruli denis $I^j_{n,m,p}$ amplitudebis gansazRvraze.

ucnobi amplitudebis gansazRvra. $A_{\alpha,\beta}$, $B_{\alpha,\beta}$, $C_{\gamma,\delta}$, $D_{\gamma,\delta}$ da $I^j_{n,m,p}$ ucnob amplitudebs gavnsazRvravT (1.2.1) da (1.2.2) sasazRvro pirobebidan. Tu am sasazRvro pirobebSi CavsvavT velebis (1.2.5) - (1.2.8) gamosaxulebebs, maSin miviRebT ucnobi amplitudebis mimarT wrfiv algebrul gantolebaTa sistemas. im SemTxvevaSi, rodesac dacemuli velis wyaro struqturis gareT imyofeba, aRniSnul sistemas eqneba saxe

$$\begin{cases}\vec{E}_2\left(\vec{r}_{\eta,\vartheta}\right)\cdot\vec{\tau}_1-\vec{E}_1\left(\vec{r}_{\eta,\vartheta}\right)\cdot\vec{\tau}_1+\vec{E}_s\left(\vec{r}_{\eta,\vartheta}\right)\cdot\vec{\tau}_1=\vec{E}_{inc}\left(\vec{r}_{\eta,\vartheta}\right)\cdot\vec{\tau}_1,\\ \vec{H}_2\left(\vec{r}_{\eta,\vartheta}\right)\cdot\vec{\tau}_1-\vec{H}_1\left(\vec{r}_{\eta,\vartheta}\right)\cdot\vec{\tau}_1+\vec{H}_s\left(\vec{r}_{\eta,\vartheta}\right)\cdot\vec{\tau}_1=\vec{H}_{inc}\left(\vec{r}_{\eta,\vartheta}\right)\cdot\vec{\tau}_1,\\ \vec{E}_2\left(\vec{r}_{\eta,\vartheta}\right)\cdot\vec{\tau}_2-\vec{E}_1\left(\vec{r}_{\eta,\vartheta}\right)\cdot\vec{\tau}_2+\vec{E}_s\left(\vec{r}_{\eta,\vartheta}\right)\cdot\vec{\tau}_2=\vec{E}_{inc}\left(\vec{r}_{\eta,\vartheta}\right)\cdot\vec{\tau}_2,\\ \vec{H}_2\left(\vec{r}_{\eta,\vartheta}\right)\cdot\vec{\tau}_2-\vec{H}_1\left(\vec{r}_{\eta,\vartheta}\right)\cdot\vec{\tau}_2+\vec{H}_s\left(\vec{r}_{\eta,\vartheta}\right)\cdot\vec{\tau}_2=\vec{H}_{inc}\left(\vec{r}_{\eta,\vartheta}\right)\cdot\vec{\tau}_2,\\ \vec{E}_s\left(\vec{r}^{\,k}_{n',m',p'}+d\vec{r}_0\right)\cdot\Delta\vec{l}^{\,k}_{n',m',p'}+\vec{E}_2\left(\vec{r}^{\,k}_{n',m',p'}+d\vec{r}_0\right)\cdot\Delta\vec{l}^{\,k}_{n',m',p'}=0,\end{cases}$$

sadac η, ϑ dieleqtrikis S zedapiris wertilis nomeria, romelSic sasazRvro piroba iwereba $(\eta,\vartheta=1,2,...)$, $k=1,2,...$.

im SemTxvevaSi, rodesac wyaro imyofeba struqturis SigniT, analogiurad miviRebT

$$
\begin{cases}
\vec{E}_2\left(\vec{r}_{\eta,\vartheta}\right)\cdot\vec{\tau}_1-\vec{E}_1\left(\vec{r}_{\eta,\vartheta}\right)\cdot\vec{\tau}_1+\vec{E}_s\left(\vec{r}_{\eta,\vartheta}\right)\cdot\vec{\tau}_1=-\vec{E}_{inc}\left(\vec{r}_{\eta,\vartheta}\right)\cdot\vec{\tau}_1,\\
\vec{H}_2\left(\vec{r}_{\eta,\vartheta}\right)\cdot\vec{\tau}_1-\vec{H}_1\left(\vec{r}_{\eta,\vartheta}\right)\cdot\vec{\tau}_1+\vec{H}_s\left(\vec{r}_{\eta,\vartheta}\right)\cdot\vec{\tau}_1=-\vec{H}_{inc}\left(\vec{r}_{\eta,\vartheta}\right)\cdot\vec{\tau}_1,\\
\vec{E}_2\left(\vec{r}_{\eta,\vartheta}\right)\cdot\vec{\tau}_2-\vec{E}_1\left(\vec{r}_{\eta,\vartheta}\right)\cdot\vec{\tau}_2+\vec{E}_s\left(\vec{r}_{\eta,\vartheta}\right)\cdot\vec{\tau}_2=-\vec{E}_{inc}\left(\vec{r}_{\eta,\vartheta}\right)\cdot\vec{\tau}_2,\\
\vec{H}_2\left(\vec{r}_{\eta,\vartheta}\right)\cdot\vec{\tau}_2-\vec{H}_1\left(\vec{r}_{\eta,\vartheta}\right)\cdot\vec{\tau}_2+\vec{H}_s\left(\vec{r}_{\eta,\vartheta}\right)\cdot\vec{\tau}_2=-\vec{H}_{inc}\left(\vec{r}_{\eta,\vartheta}\right)\cdot\vec{\tau}_2,\\
\vec{E}_s\left(\vec{r}^{\,k}_{n',m',p'}+d\vec{r}_0\right)\cdot\Delta\vec{l}^{\,k}_{n',m',p'}+\vec{E}_2\left(\vec{r}^{\,k}_{n',m',p'}+d\vec{r}_0\right)\cdot\Delta\vec{l}^{\,k}_{n',m',p'}=-\vec{E}_{inc}\left(\vec{r}^{\,k}_{n',m',p'}+d\vec{r}_0\right)\cdot\Delta\vec{l}^{\,k}_{n',m',p'}.
\end{cases}
$$

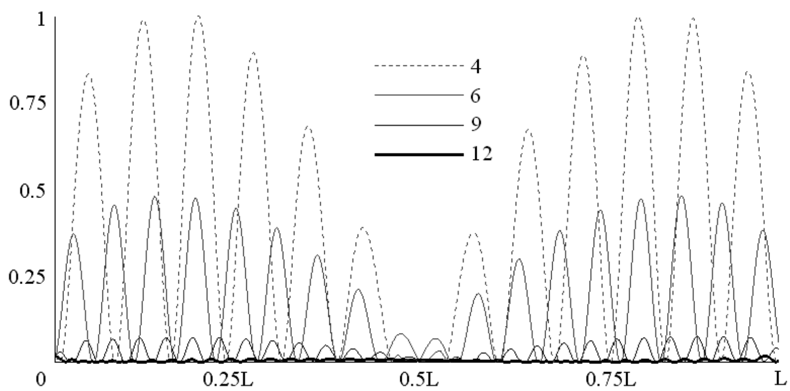
am sistemebis amoxsna kompiuteris saSualebiT xdeba, ris Sedegad vpoulobT damxmare wyaroebis amplitudebs da aseve denis amplitudebs mesris elementebis gaswvriv. amis Semdeg vpoulobT ucnob gare da Sida velebs.

$1.3 ricxviTi eqsperimentis Sedegebi

ricxviT eqsperimentebamde, pirvel rigSi, Semowmda Sesabamisi algoriTmis sizuste da miRebuli amonaxsnebis krebadoba.

krebadobis da amoxsnis sizustis Semowmeba. damxmare gamomsxivebleebis meTodis gamoyenebisas mniSvnelovania damxmare parametrebis optimaluri SerCeva. damxmare parametrebs warmoadgenen: segmentis sigrZe mesris elementSi da elementis radiusi dacemuli talRis λ sigrZesTan SedarebiT, damxmare zedapiris daSoreba sxeulis realur zedapiridan, aseve kolokaciis wertilebis raodenoba λ^2 farTobze. am damxmare parametrebis mniSvnelobebzea damokidebuli miRebuli amonaxsnis krebadoba da sizuste. maTi optimaluri mniSvnelobebis codna xels uwyobs rTuli struqturebis eleqtromagnituri Tvisebebis saTanado gamokvlevas ricxviTi eqsperimentebis saSualebiT.

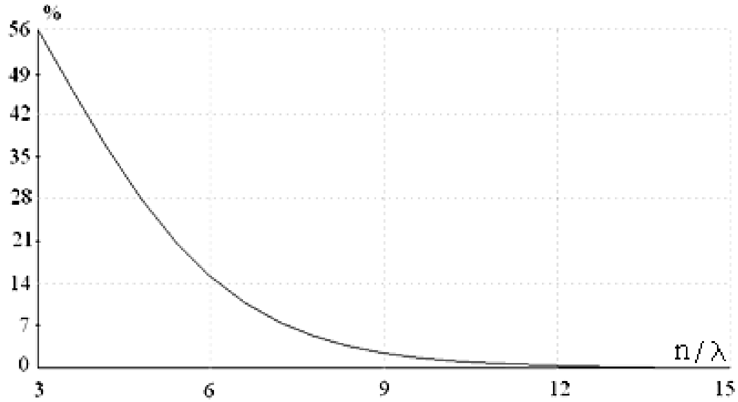
qvemoT moyvanili naxazi (nax. 1.3.1) gviCvenebs, Tu rogoraa damokidebuli sasazRvro pirobebis Sesruleba mesris elementis gaswvriv kolokaciis wertilebis raodenobaze. gadaxris yvela mniSvneloba danormirebulia maqsimalur mniSvnelobaze, romelic miiReba oTxi kolokaciis wertilis SemTxvevaSi talRis sigrZeze.



nax. 1.3.1 cdomilebis damokidebuleba kolokaciis wertilebis raodenobaze

rogorc am naxazidan Cans, gadaxra sasazRvro pirobis Sesrulebidan kolokaciis wertilebs Soris mcirdeba maTi raodenobis gazrdisas. optimaluri damxmare parametrebis Ziebam gviCvena, rom saukeTeso miaxloveba eleqtrulad wvril mavTulTan miiRweva, rodesac misi dr_0 radiusi imyofeba 0.03λ mniSvnelobis farglebSi.

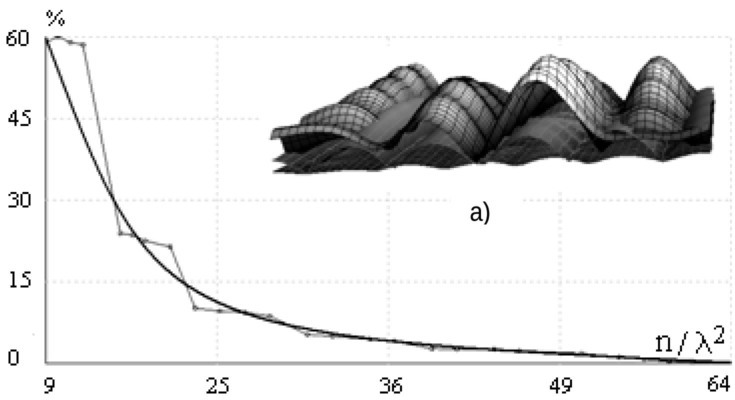
Semdegi naxazi (nax. 1.3.2) gviCvenebs amonaxsnis maqsimaluri cdomilebis damokidebulebas n/λ sidideze, romelic warmoadgens kolokaciis wertilebis raodenobas talRis sigrZeze.



nax. 1.3.2 cdomilebis damokidebuleba n/λ sidideze

iTvleba, rom cdomileba romelic Seesabameba romelime konkretul n/λ mniSvnelobas, aris Tanabari mTeli elementis (mavTulis) gaswvriv. es imas niSnavs, rom realuri cdomileba am mniSvnelobaze ufro naklebia. magaliTad, cxra kolokaciis wertilis SemTxvevaSi cdomileba or procentze naklebs Seadgens.

naxazebi 1.3.3 a) da b) gviCveneben sasazRvro pirobebis Sesrulebis sizustes $\vec{E}$ veqtorisaTvis dieleqtrikis zedapirze, kolokaciis wertilebis sxvadasxva raodenobaze.



b)

nax. 1.3.3 cdomilebis damokidebuleba kolokaciis wertilebis raodenobaze dieleqtrikis zedapiris gaswvriv

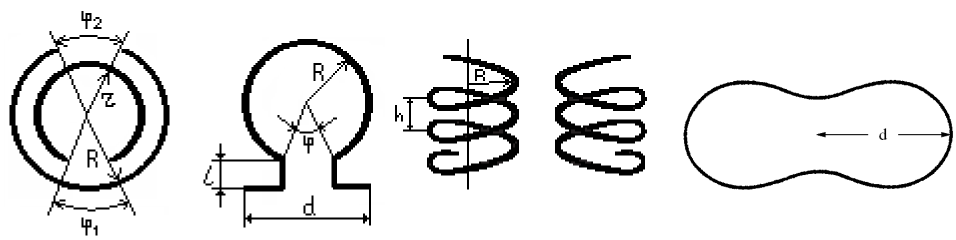
gamoTvlis mrude texilia (b)), radgan rTulia mivaRwioT mis sigluves. 25 kolokaciis wertili λ^2 farTobze gvaZlevs 10% cdomilebas. 36 wertilis SemTxvevaSi cdomileba 3% Seadgens. aseve, cdomileba

Seadgens mxolod 1%, rodesac gagvaCnia 64 kolokaciis wertili, rac Seesabameba rva wertils talRis sigrZeze. dieleqtrikis wiboebi da wveroebi Canacvlebulia cilindruli da sferuli zedapirebiT da maTi (momrgvalebis) radiusia $r_0 \approx 0.03\lambda$. amave dros, momrgvalebis radiusis es mniSvneloba gansazRvravs damxmare zedapiris maqsimalur daSorebas dieleqtrikis realur zedapiridan. dieleqtrikis zedapiri ar Seicavs Cazneqil nawilebs da amitom gare damxmare zedapiri SeiZleba iqnas misgan daSorebuli talRis sigrZis rigis manZilze. Sida damxmare zedapiris aRebuli daSoreba udris momrgvalebis radiuss. rac Seexeba damxmare gamomsxiveblebs, isini warmoadgenen hiugensis wyaroebs da maTi gamosxiveba mimarTulia aRsaweri velisaken.

Semdeg moyvanilia ricxviTi Sedegebi, romelnic miRebulia zemoaRniSnuli cdomilebebis gaTvaliswinebiT. es cdomileba saSualod 2-5% farglebSia. rezonansuli sixSireebis SemTxvevaSi cdomileba matulobs, magram is ar aRemateba 10%. cnobilia, rom praqtikaSi, nebismieri fizikuri sididis gazomva garkveuli sizustiT xdeba. amitom miRebuli ricxviTi Sedegebi Seesabamebian odnav wanacvlebul parametebis mqone struqturebs.

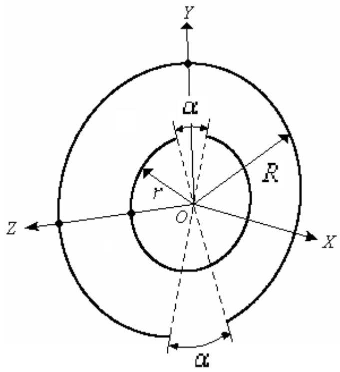
yvela moyvanili ricxviTi Sedegi rCeba samarTliani farTo sixSirul areSi, romlis farglebSic SeiZleba gamoyenebul iqnas maqsvelis gantolebaTa sistema. amitom, gamokvleuli struqturebis geometriuli parametrebi moyvanilia dayvanil, talRis sigrZis erTeulebSi.

ganixileba rezonansuli elementis oTxi gansxvavebuli forma: ori koncentrirebuli Ria rgoli, berZnuli aso Ω formis elementi, kiraluri elementi (spirali) da aseve elementi, romelsac gaaCnia kasinis mrudis forma (nax. 1.3.4). yoveli elementi SeiZleba warmodgenil iqnas, rogorc maRali vargisianobis mqone rxeviTi konturi, romelsac gaaCnia sakuTari induqtioba, tevadoba da winaRoba.



nax. 1.3.4 mesris elementis magaliTebi

ori koncentrirebuli Ria rgolis eleqtrodinamikuri Tvisebebis gamokvleva. aq gamokvleulia im erTerTi SesaZlo elementis rezonansuli Tvisebebi, romelic SeiZleba iqnas gamoyenebuli kompleqsuri masalis misaRebad. kerZod, iseTi masalis, romelsac uaryofiTi gardatexis maCvenebeli gaaCnia.

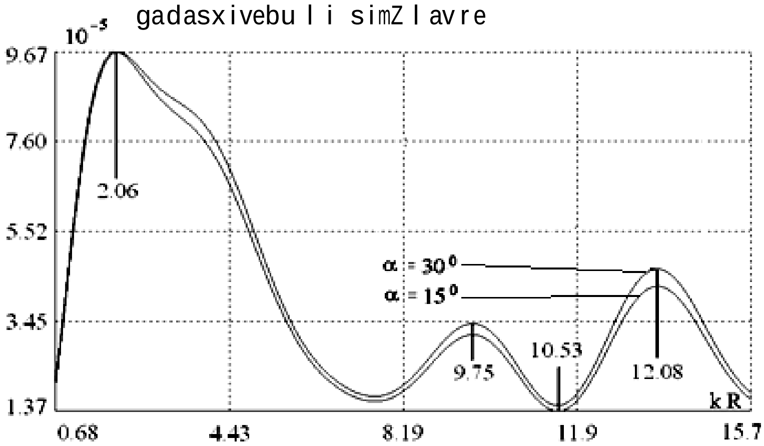


nax.F1.3.5 rezonansuli elementis geometria

es rezonansuli elementi warmoadgens R da $r = 0.5R$ radiusebis mqone koncentrirebul or Ria rgols [37]. orive rgolis Ria seqtori α kuTxes Seadgens da am seqtorebs urTierTsawinaaRmdego orientacia gaaCniaT (nax. 1.3.5).

ganxilul elements ecema brtyeli erTeulovani amplitudis mqone talRa, romelic OX RerZis gaswvriv vrceldeba da polarizebulia OY RerZis gaswvriv. naxazze 1.3.6 moyvanilia gadasxivebuli simZlavris damokidebuleba $kR = 2\pi R/\lambda$ sidideze or gansxvavebul SemTxvevaSi, rodesac rgolebis Ria seqtoris α kuTxe Seadgens 15^0 da 30^0.

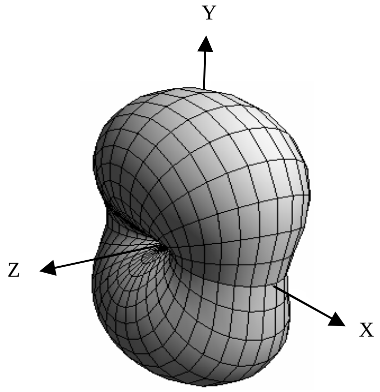
gadasxivebuli simZlavre iqna napovni poitingis veqtoris integrirebiT ganxiluli elementis garSemo agebuli sferos zedapiris gaswvriv. am grafikis pikebi Seesabamebian rezonansis SemTxvevas, radgan rezonansul sixSireebze aRznebuli denis amplituda izrdeba da Sesabamisad izrdeba gadasxivebuli simZlavre. gansxvaveba gadasxivebuli simZlavreebs Soris am or SemTxvevaSi Cndeba meore ($kR = 9.75$) da mesame ($kR = 12.8$) rezonansul sixSireebze.



nax. 1.3.6 sixSiruli maxasiaTebeli

pirveli rezonansis dros gadasxivebuli veli 5, 6-jer ufro maRalia. aq adgili aqvs ormag rezonanss, radgan elementi or nawilisgan Sedgeba. amiT aixsneba is, Tu ratom aris pirveli rezonansi ufro farTo vidre danarCeni rezonansebi.

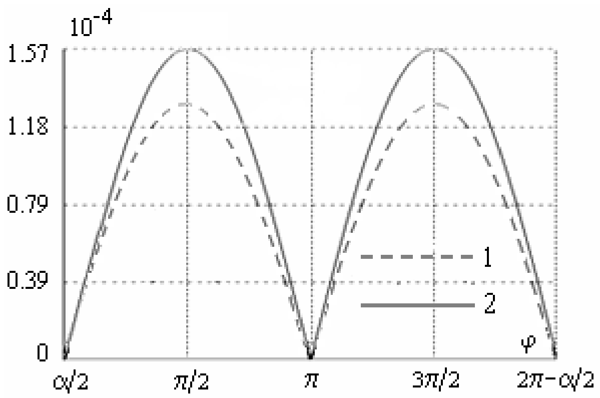
naxazze 1.3.7 moyvanilia ganxiluli elementis mier gadasxivebuli velis diagrama, rodesac mas ecema rezonansuli sixSiris mqone brtyeli talRa. moyvanilia aseve Sesabamisi parametrebi.



gare rgolis radiusi - R
Sida rgolis radiusi - $0.5R$
Ria seqtoris kuTxe - 15^0
kR=2.06

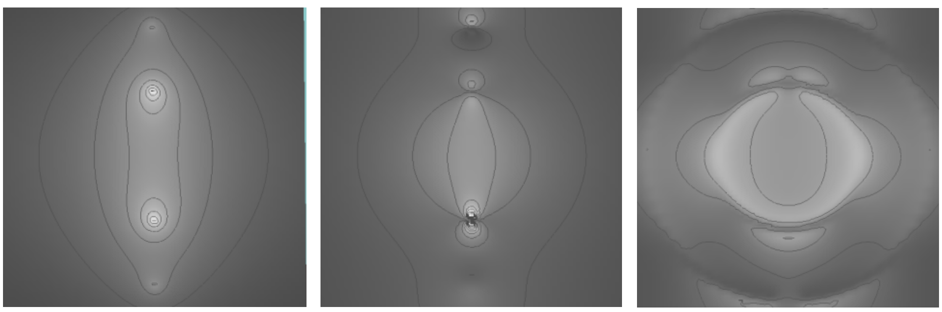
nax. 1.3.7 Sori velis diagrama rezonansul sixSireze

ganxilul elements maRali vargisianoba gaaCnia. sixSiris cvlilebiT rezonansis maxlobload, Segvlilia gavzardoT an SevamciroT denis amplituda gare an Sida rgolSi. Semdeg naxazze (nax. 1.3.8) moyvanilia is SemTxveva, rodesac ufro maRali amplitudis deni aRiZvreba Sida rgolSi.



nax. 1.3.8 denebis ganawileba elementis rgolebSi

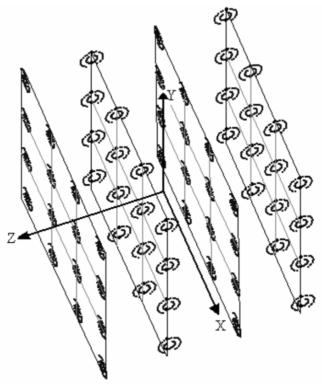
es aseve kargad Cans 1.3.9 naxazze sadac moyvanilia axlo velis ganawileba rezonansul sixSireze a) XOZ, b) XOY, da g) YOZ sibrtyeebSi. Sida rgolis garSemo ufro maRali veli formirdeba.



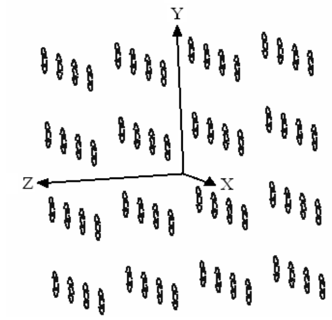
a) b) g)

nax. 1.3.9 axlo velis ganawileba a) XOZ, b) XOY da g) YOZ sibrtyeSi

Semdeg ganxilulia aseTi tipis elementebisagan Semdgari perioduli struqturebi Tavisufal sivrceSi, elementebis gansxvavebuli orientaciis dros (nax. 1.3.10, 1.3.11).

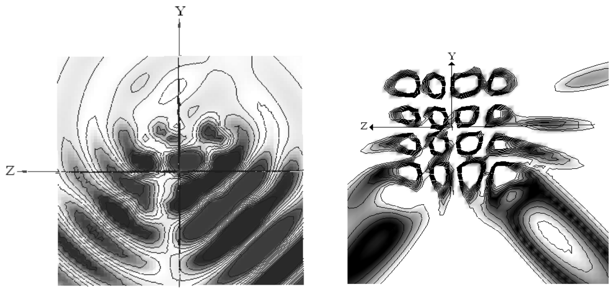


nax. 1.3.10 mesris periodebi: $d_1 = d_2 = d_3 = 0.5\lambda$, rgolebis radiusebi: $R = 0.08\lambda$, $r = 0.04\lambda$



nax. 1.3.11 mesris periodebi: $d_1 = d_2 = d_3 = 0.8\lambda$, rgolebis radiusebi: $R = 0.14\lambda$, $r = 0.07\lambda$

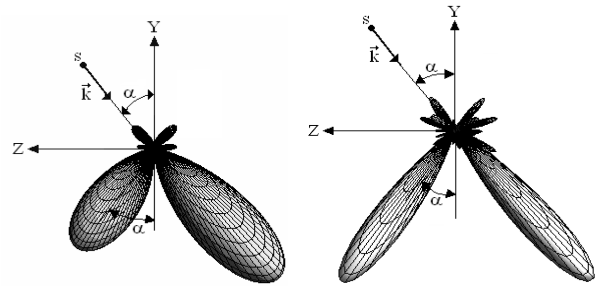
manZili elementebs Soris Seesabameba ormag rezonanss da elementebis raodenobaa 4×4×4. aseTi tipis struqturebi kompleqsur Tvisebebs amJRavneben. naxazebi 1.3.12 a) da b) gviCveneben axlo velis ganawilebas am ori gansxvavebuli struqturis SemTxvevaSi.



a) b)

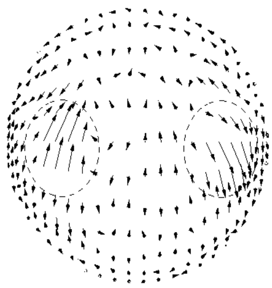
nax. 1.3.12 axlo velis ganawileba a) 1.3.10 da b) 1.3.11 geometriis SemTxvevaSi

Semdeg moyvanilia ganxiluli struqturebis mier gamosxivebuli Sori velis diagramebi (nax. 1.3.13 a) da b)). rogorc vxedavT, orive SemTxvevaSi miiReba ori didi ZiriTadi foToli. erT maTgans gaaCnia dacemuli velis mimarTuleba, xolo meore foToli Seesabameba uaryofiT gardatexas.



a) b)

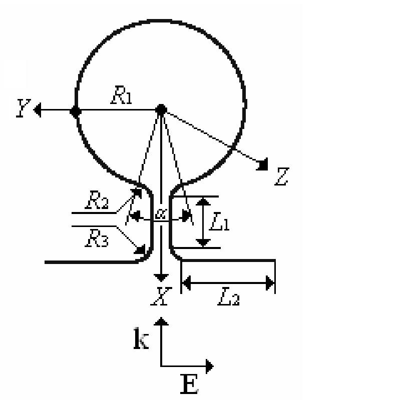
nax. 1.3.13 Sori velis diagrama a) 1.3.10 da b) 1.3.11 geometriis SemTxvevaSi



nax. 1.3.14 velis veqtorebis ganawileba sferul zedapirze

gabneuli velis polarizaciuli Tvisebebis Seswavlam gviCvena, rom am ZiriTadi foTlebis gaswvriv mas gaaCnia urTierTsawinaaRmdego mbrunavi polarizacia (nax. 1.3.14). es ukanaskneli ufro ukeTesad Cans kompiuteruli animaciis dros.

Ω elementis zogierTi eleqtrodinamikuri Tvisebebis gamokvleva. qveviT moyvanilia sxva rezonansuli elementis gamokvlevis Sedegebi. elementi warmoadgens wvril gamtars da gaaCnia berZnuli aso Ω forma. ganixileba SemTxvevebi rodesac igi imyofeba Tavisufal sivrceSi da aseve sasruli zomebis mqone dieleqtrikSi. aseTi saxis elementis ganxilva sainteresoa imiT, rom mis bazaze SeiZleba Seiqmnas mimarTuli gamosxivebis mqone antenuri mowyobiloba. amasTanave aseTi elementi sainteresoa imitom, rom Tu dacemuli veli polarizebulia elementis horizontaluri “fexebis” gaswvriv, maSin mis momrgvalebul nawilSi maRali amplitudis deni aRiZvreba. es deni aCens maRal $\vec{H}$ magnitur da Sesabamisad maRal $\vec{E}$ eleqtrul velebs. es $\vec{E}$ da $\vec{H}$ velebi urTierTmarTobulni arian. Tu saTanadod SevarCevT Ω elementis parametrebs maSin igi hiugensis gamomsxivebelis msgavsi iqneba. naxazze 1.3.15 moyvanilia Ω elementis geometria da aseve misi parametrebi, romelnic iyvnen SerCeulni farTo sixSiruli maxasiaTebelis misaRebad (nax. 1.3.16).

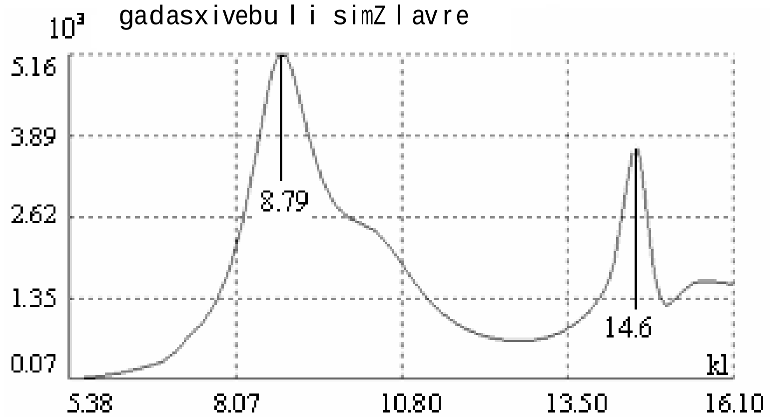


l- elementis sruli sigrZe

$R_1 = 0.1l$, $R_2 = 0.02l$, $R_3 = 0.02l$,

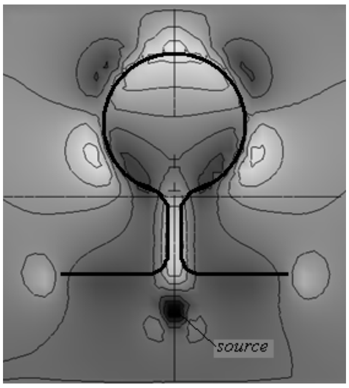
$L_1 = 0.05l$, $L_2 = 0.1l$, $\alpha = \pi/12$

nax. 1.3.15 Ω- elementi da misi parametrebi

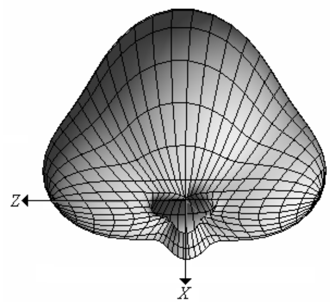


nax. 1.3.16 gadasxivebuli simZlavris damokidebuleba kl sidideze

Semdeg naxazebze moyvanilia axlo velis ganawileba da Sori velis diagrama rodesac dacemuli velis wertilovani gamomsxivebeli elementis maxloblad imyofeba (nax. 1.3.17, 1.3.18). imisaTvis, rom avaridoT Tavi velis singularobebs elementis gaswvriv, velis daxatvis sibrtye mcire manZiliTaa daSorebuli elementis sibrtyisgan. Sori velis diagrama gviCvenebs, rom gadasxivebuli energiis umetesi nawili erTi mimarTulebiT vrceldeba.

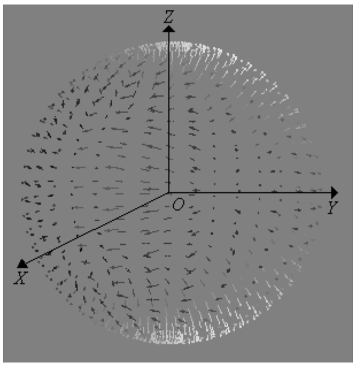


nax. 1.3.17 ax l o ve l is ganawi l eba omega e l emen t is SemTxvevaSi

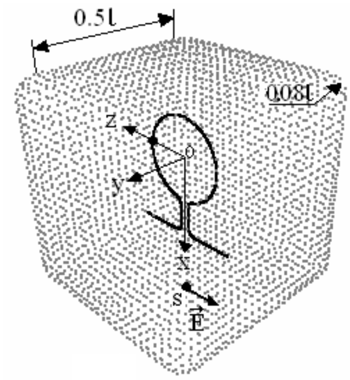


nax. 1.3.18 So r i ve l is d iag r ama

Semdeg moyvani l ia gabneu l i ve l is po l a r izac iis ganawi l eba s f e r u l zedapi r ze So r z onaSi (nax. 1.3.19). misma ana l izma gviCvena, r om gabneu l ve l s ga rkveu l i mima r Tu l ebebiT gaaCnia e l i f su r i da w r iu l i po l a r i zac ia.



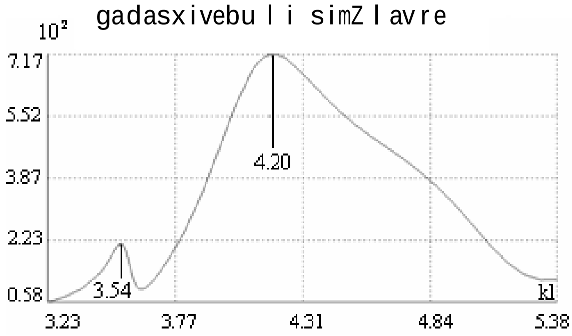
nax. 1.3.19 po l a r i zac iis ganawi l eba So r z onaSi



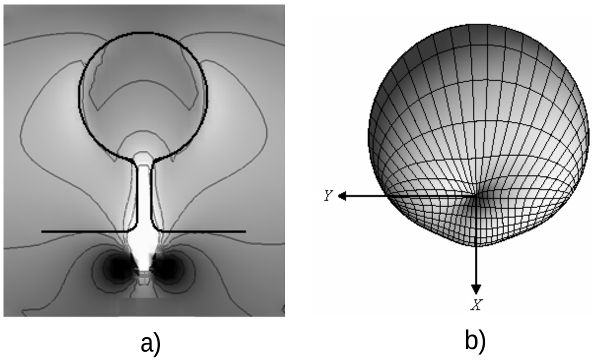
nax. 1.3.20 Ω-e l emen t i d ie l eq t r iku l i kubis SigniT

imisaTvis r om miviRoT r ezonansi u f r o fa r To sixSi r u l diapaz onSi, Ω e l emen t i ganixi l eba $\varepsilon = 4$ SeRwevad obis mqone d ie l eq t r ikis SigniT. d ie l eq t r iks gaaCnia g l uvi kubis f o r ma (nax. 1.3.20). dacemu l i ve l is we r t i l ovani wya r o am d ie l eq t r ikis SigniT imyo f eba.

Semdeg naxaz ze moyvani l ia sis t emis mie r gadasxivebu l i simZ l av r is damokidebu l eba kl pa r ame t r ze (nax. 1.3.21). fa r To r ezonansi $kl = 4.2$ mniSvne l obaze a r is o r magi, anu wa r moadgens r ezonanss Ω e l emen tsa da d ie l eq t r iks So r is. naxazebi 1.3.22 a) da b) gviCveneben ax l o ve l is ganawi l ebas da So r i ve l is d iag r amas. am SemTxvevaSi gamosxivebu l ve l s a r gaaCnia gamokveTi l i mima r Tu l eba.

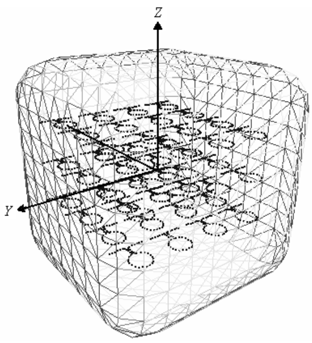


nax. 1.3.21 gadasxivebuli simZlavris damokidebuleba kl sidideze dieleqtrikis SemTxvevaSi

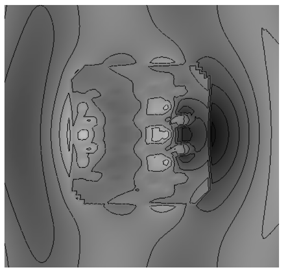


nax. 1.3.22 a) axlo velis ganawileba, b) Sori velis diagrama

Semdeg ganxilul iqna Ω elementebisagan Semdgari samganzomilebiani perioduli meseri, romelic iyo moTavsebuli dieleqtrikSi (nax. 1.3.23). elementebis raodenoba meserSi tolia $4\times4\times3$ da manZili maT Soris udris $4R_1$, sadac R_1 omega elementis rgolis radiusia. naxazze 1.3.24 moyvanilia axlo velis ganawilebas XOZ sibrtyeSi. rogorc Cans energiis umetesi nawili garkveuli mimarTulebiT vrceldeba.

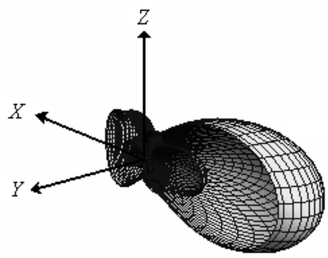


nax. 1.3.23 omega elementebisagan Semdgari meseri dieleqtrikSi



nax. 1.3.24 axlo velis ganawileba XOZ sibrtyeSi

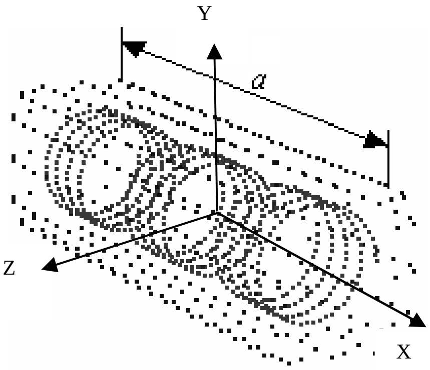
naxazze 1.3.25 moyvanilia Sori velis diagramebis Sedareba rodesac gagvaCnia 1 da $4\times4\times3$ Ω elementi. gadasxivebuli velis maqsimaluri mniSvneloba $4\times4\times3$ elementis SemTxvevaSi gacilebiT ufro metia radgan yoveli elementi rezonansSi imyofeba da Sedegad miiReba maTi velebis superpozicia.



nax. 1.3.25 Sori velis diagrama 1 da $4\times4\times3$ elementebis SemTxvevaSi

spiralebisgan Semdgari metalo - dielqtrikuli struqturebi. Semdeg gamokvleulia iseTi struqtura, romelsac SeuZlia wrfivad polarizebuli talRa gardaqmnas elifsurad polarizebulSi. amasTanave mis mier gamosxivebul vels gaaCnia brtyeli diagrama garkveuli sxeulovani kuTxis farglebSi [38, 39] (SedarebiT ufro adre ganxilul iqna aseTi amocanis organzomilebiani SemTxveva [40]). struqtura warmoadgens sam spirals, romelnic imyofebian dieleqtrikSi. ganixileba ori SemTxveva: pirvel SemTxvevaSi dieleqtriks gaaCnia cilindris forma da orive mxridan SemosazRvrulia naxevarsferoebiT. meore SemTxvevaSi dieleqtriks gaaCnia paralelepipedis forma. dacemuli velis kombinirebuli gamomsxivebeli imyofeba struqturis SigniT erTerTi spiralis maxlobad.

Semdegi naxazi gviCvenebs struqturis geometrias paralelepipedis fromis SemTxvevaSi (nax. 1.3.26 a)). aq aseve moyvanilia parametrebis is mniSvnelobebi, rodesac am struqturas gaaCnia zemoaRniSnuli Tvisebebi (nax. 1.3.26 b)).

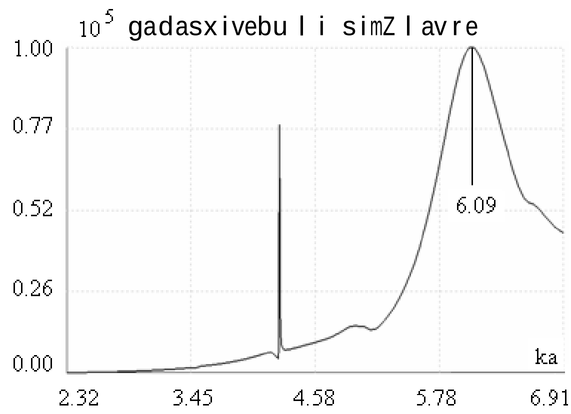


a)

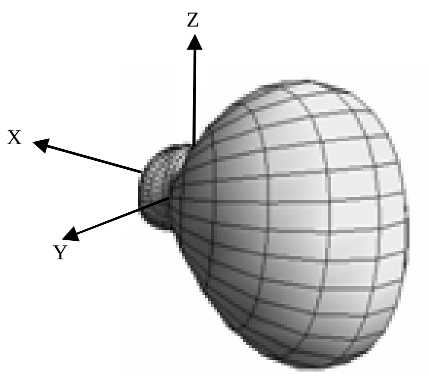
dieleqtrikuli paralelepipedis zomebi X, Y, Z RerZebis gaswvriv: $a\times0.25a\times0.25a$; dieleqtrikis kuTxeebis momrgvalebis radiusi $0.055a$; dieleqtrikuli SeRwevadoba 4; talRis sigrZe dieleqtrikis gareT a; spiralis simaRle $0.18a$; spiralis radiusi $0.09a$; manZili spiralebs Soris $0.32a$; spiralebis raodenoba 3; kombinirebuli dipolis koordinata X RerZze - $0.43a$; kombinirebuli dipolis polarizacia: Y - eleqtruli dipolisaTvis, Z - magniturisaTvis.

b)

nax. 1.3.26 a) struqturis geometria, b) struqturis parametrebi



a)

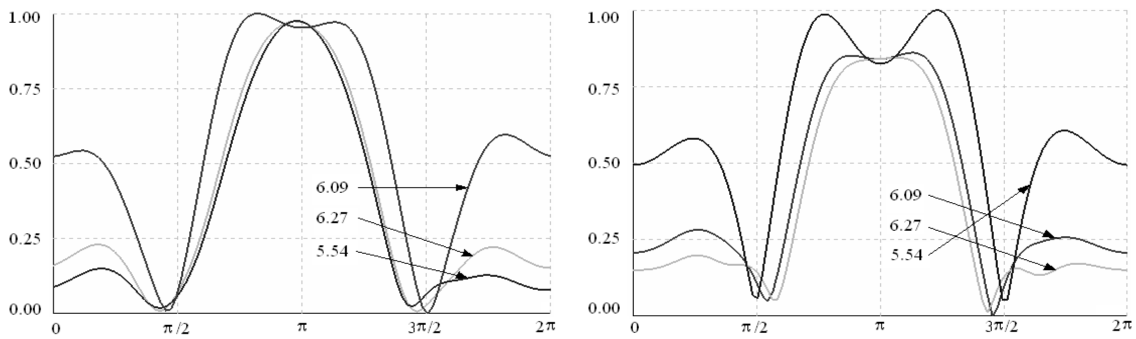


b)

nax. 1.3.27 a) struqturis sixSiruli maxasiaTebeli, b) Sori velis diagrama

naxazze 1.3.27 a) moyvanilia ganxiluli struqturis mier gadasxivebuli simZlevaris damokidebuleba *ka* parametrze. am parametris moyvanil farglebSi, struqturas gaaCnia ori rezonansi. pirveli rezonansi metad viwroa da amitom interess meore rezonansi warmoadgens, rodesac ka=6.09. naxazze 1.3.27 b) moyvanilia Sori velis diagrama meore rezonansis SemTxvevaSi. ganxiluli struqturis mier gamosxivebuli energiis ZiriTadi nawili vrceldeba OX RerZis sawinaaRmdego mimarTulebiT.

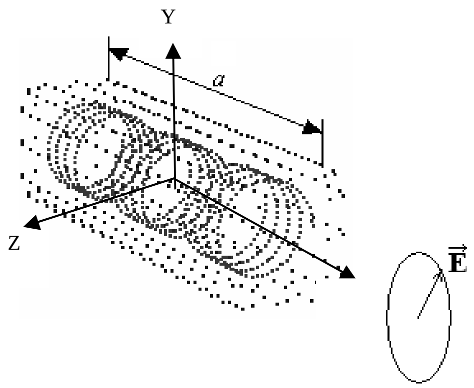
Semdeg moyvanilia Sori velis diagrama or urTierTmarTobul sibrtyeSi rezonansis dros da aseve mis maxloblad (nax. 1.3.28 a), b)). mniSvnelobebi danormirebulia maqsimalur mniSvnelobaze. rogorc Cans ganxiluli struqturis mier gamosxivebul vels gaaCnia erTi da igive amplituda garkveul kuTxur diapazonSi da am diapazonSi Sori velis diagrama brtyelia.



a) b)

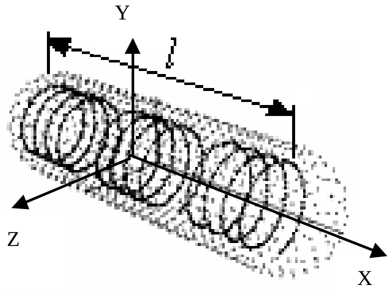
nax. 1.3.28 Sori velis diagrama a) XOY da b) XOZ sibrtyeSi

rezonansul sixSireebze dieleqtrikSi moTavsebul spiralebSi aRiZvreba maRali amplitudis denebi, ris Sedegadac gabneul vels elifsuri polarizacia gaaCnia (nax. 1.3.29).



nax. 1.3.29 elifsurad polarizebuli talRa struqturis gareT

Semdeg ganxilul iqna SemTxveva, rodesac dieleqtriks cilindruli forma gaaCnia. naxazze 1.3.30 moyvanilia ganxiluli struqturis geometria da misi parametrebis mniSvnelobebi, rodesac mas analogiuri Tvisebebi gaaCnia.

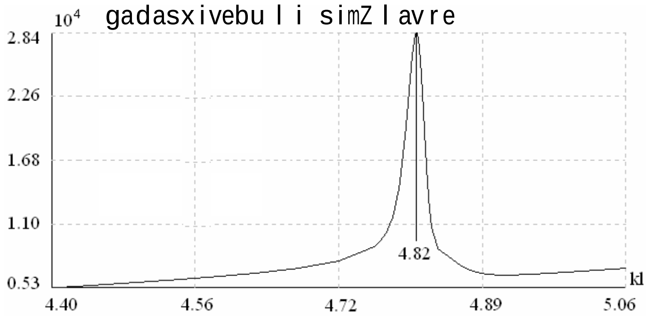


dieleqtrikuli cilindris radiusi $0.183l$; dieleqtrikuli SeRwevadoba 4; talRis sigrZe dieleqtrikis gareT $0.183l$; spiralis simaRle $0.27l$; spiralis radiusi $0.13l$; manZili spiralebs Soris $0.4l$; spiralebis raodenoba 3; kombinirebuli dipolis koordinata OX RerZze: $0.67l$; kombinirebuli dipolis polarizacia: Y - eleqtruli dipolisaTvis, Z - magnituri dipolisaTvis

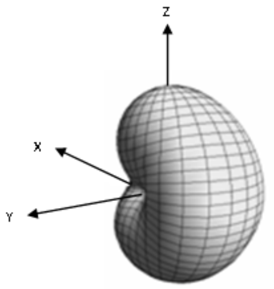
a) b)

nax. 1.3.30 a) struqturis geometria, b) struqturis parametrebi

Semdegi naxazi gviCvenebs gadasxivebuli simZlavris damokidebulebas kl parametrze, sadac l cilindris simaRlea (nax. 1.3.31). ganxiluli struqturis Tvisebebi Sesawavlilia $kl = 4.82$ rezonansis maxloblad. aq aseve moyvanilia Sori velis diagrama rezonansul sixSireze (nax. 1.3.32). rogorc Cans gadasxivebuli velis energiis ZiriTadi nawili vrceldeba garkveuli mimarTulebiT, xolo mis sawinaaRmdego mimarTulebiT veli faqtiurad qreba.

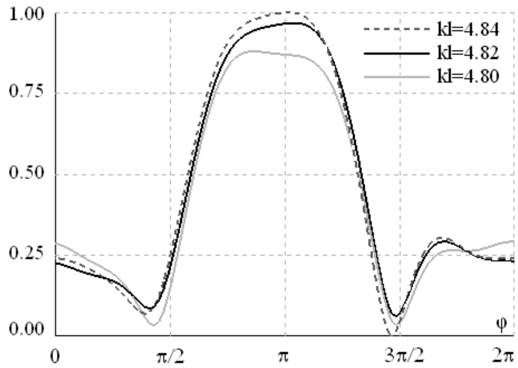


nax. 1.3.31 sixSiruli maxasiaTebeli

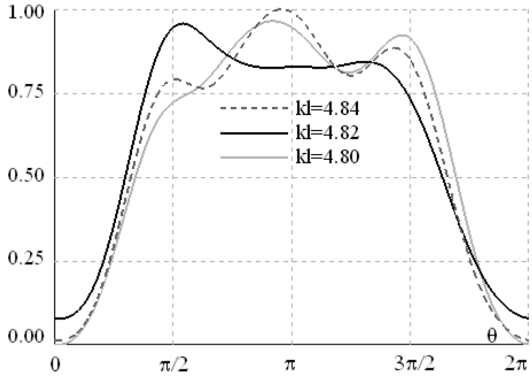


nax. 1.3.32 Sori velis diagrama

Semdeg naxazze moyvanilia Sori velis diagrama orurTierTmarTobul sibrtyeSi rezonansis SemTxvevaSi da aseve rezonansis maxloblad (nax. 1.3.33). rogorc Cans, Sor zonaSi vels gaaCnia Tanabari amplitudis mniSvnelobebi garkveul kuTxur diapazonSi da Sesabamisad sakmaod brtyeli diagrama.

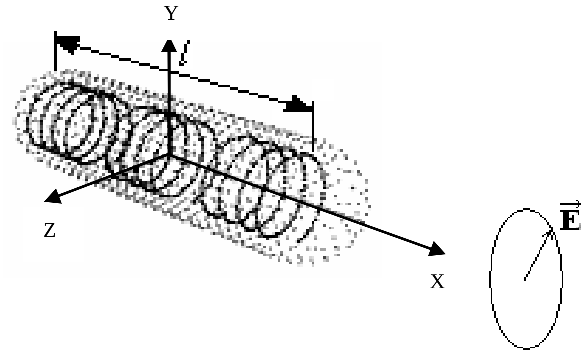


a)



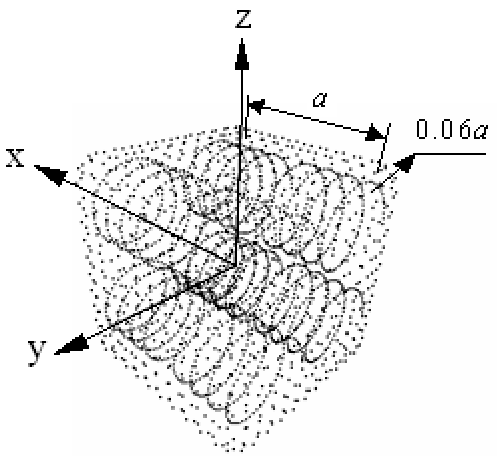
b)

nax. 1.3.33 Sori velis diagrama a) XOY da b) XOZ sibrtyeSi



nax. 1.3.34 elifsurad polarizebuli
talRa struqturis gareT

rezonansis SemTxvevaSi, denis amplituda spiralebSi mkveTrad izrdeba da mTeli struqturis mier gamosxivebul vels elifsuri polarizacia gaaCnia (nax. 1.3.34).

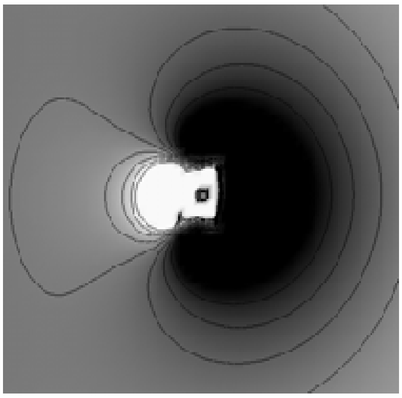


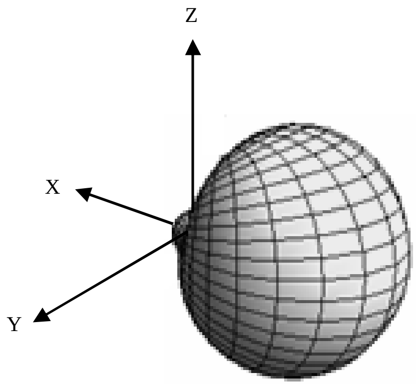
spiralis radiusi $0.2a$;
spiralebis raodenoba 8;
spiralis simaRle $0.4a$
manZili spiralebs Soris $0.5a$

a) b)

nax. 1.3.35 a) struqturis geometria, b) struqturis parametrebi

Semdeg iqna ganxiluli perioduli meseri, romelic Sedgeba 2×2×2 spiralisagan da aris moTavsebuli gluvi dieleqtrikuli kobis SigniT, romlis SeRwevadobaa $\varepsilon = 4$ (nax. 1.3.35). dacemuli velis kombinirebuli wyaro imyofeba dieleqtrikis centrSi da asxivebs OX RerZis sawinaaRmdego mimarTulebiT. gamosxivebul vels gaaCnia mbrunavi polarizacia da axlosaa wriulTan. qvemoT moyvanilia axlo velis ganawileba da Sori velis diagrama marTkuTxa sakoordinato sistemaSi (nax. 1.3.36 da 1.3.37). rogorc vxedavT, diagramas gaaCnia Π asos forma didi sxeulovani kuTxis farglebSi. energiis umetesi nawili vrceldeba mxolod erT naxevarsivrceSi.

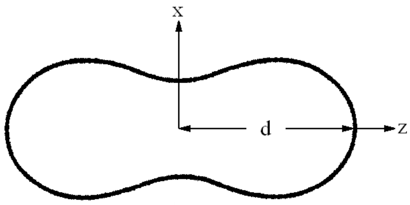




nax. 1.3.36 axlo velis ganawileba XOZ sibrtyeSi

nax. 1.3.37 Soris velis diagrama

kasinis elementis SemTxveva. aq ganixileba SemTxveva rodesac mesris elements kasinis ovalis forma gaaCnia (nax. 1.3.38) [41, 42].



nax. 1.3.38 kasinis elementi

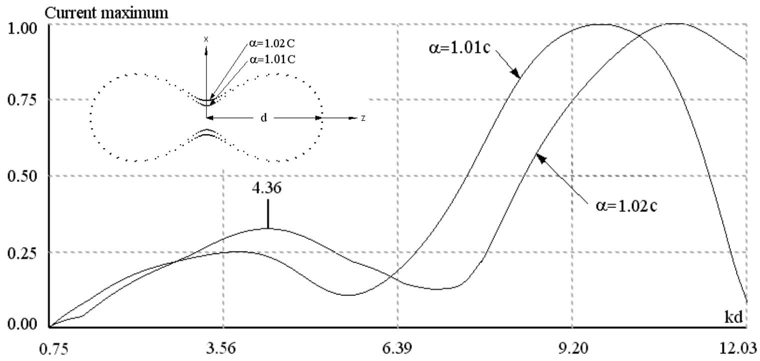
polarul koordinatebSi am mrudis gantolebaa

$$\rho(\varphi)=\sqrt{c^2\cos 2\varphi+\sqrt{a^4-c^4\sin^2 2\varphi}}\ ,$$

sadac c fokusebis Soris manZilis naxevaria, xolo a garkveuli ricxvia. Cven ganvixilavT SemTxvevas, rodesac sruldeba utoloba $c<a<\sqrt{2}c$ da kasinis mruds oTxi gadaRunvis wertili gaaCnia.

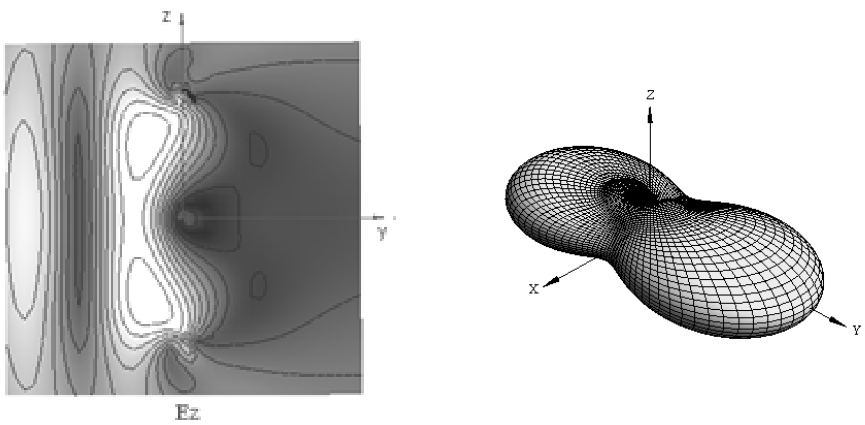
aseTi saxis elementis ganxilva sainteresoa imiT, rom mas farTo rezonansebi gaaCnia. es xels Seuwyobs Sesabamisi struqturebis kompleqsuri Tvsebebis gamovlenas farTo sixSirul diapazonSi.

erTi elementis SemTxveva brtyeli dacemuli talRis dros. elementi imyofeba XOZ sibrtyeSi, dacemuli veli warmoadgens brtyel talRas, romelic vrceldeba OY RerZis gaswvriv da gaaCnia orive OX da OZ polarizacia. qvemoT moyvanili suraTi gviCvenebs aRZruli denis danormirebuli maqsimumis damokidebulebas kd parametrze, sadac k talRuri ricxvia, xolo d warmoadgens kasinis maqsimalur radiuss (nax. 1.3.39). aq moyvanilia ori gansxvavebuli mrudi, romlebic Seesabamebian elementis sxvadasxva geometrias ($a=1{,}01c$ da $a=1{,}02c$). am mrudebis maqsimumebi miuTiTeben rezonansis SemTxvevas. a koeficientis umniSvnelo gazrdam gamoiwvia rezonansis wanacvleba marjvniv.



nax. 1.3.39 gadasxivebuli simZlavris
damokidebuleba *kd* parametrze

Semdeg moyvanilia axlo velis E_z komponentis ganawileba *YOZ* sibrtyeSi da aseve Sori velis diagrama rezonansis SemTxvevaSi, rodesac *kd*=4.36, $a=1,02c$ (nax. 1.3.40 a) da b)).



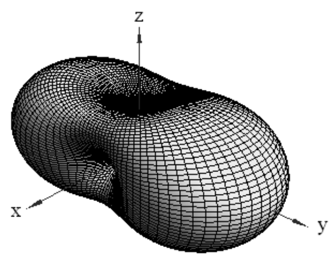
a) b)

nax. 1.3.40 a) axlo velis ganawileba,
b) Sori velis diagrama

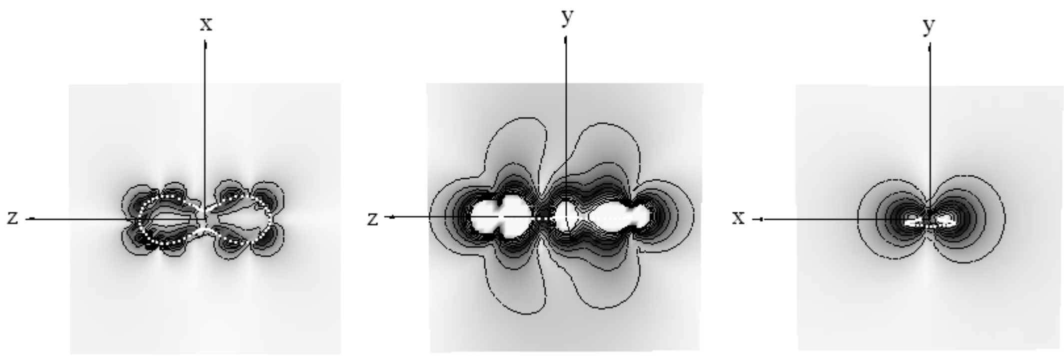
grafikis Semdegi piki maRali rigis rezonanss warmoadgens da Sori velis diagramas mravali foToli gaaCnia. velis Tvisebebis gamokvlevam gviCvena, rom mas elifsuri polarizacia gaaCnia da elifsurobis koeficientia 1:2.5.

erTi elementis SemTxveva rodesac wertilovani dacemuli velis wyaro masTan axlos imyofeba. elementi kvlav *XOZ* sibrtyeSi imyofeba. mas ecema *X* polarizaciis mqone talRa, romlis wertilovani gamomsxivebeli imyofeba wertilSi (0, -3*d*, 0). qveviT moyvanilia danormirebuli gadasxivebuli simZlavris damokidebuleba *kd* sidideze (nax. 1.3.41). moyvanil farglebSi Cven vxedavT or piks, romelnic rezonanss Seesabamebian. gamokvleulia orive rezonansi. aq moyvanilia Sori velis diagrama (nax. 1.3.42) da axlo velis *Ez* komponentis ganawileba rodesac *kd*=1.79 (nax. 1.3.43). rogorc vxedavT, gamokveTilia energiis gavrcelebis ori mimarTuleba - *OY* RerZis gaswvriv da mis sawinaaRmdegod.

nax. 1.3.41 gadasxivebuli simZlavris damokidebuleba *kd* sidideze

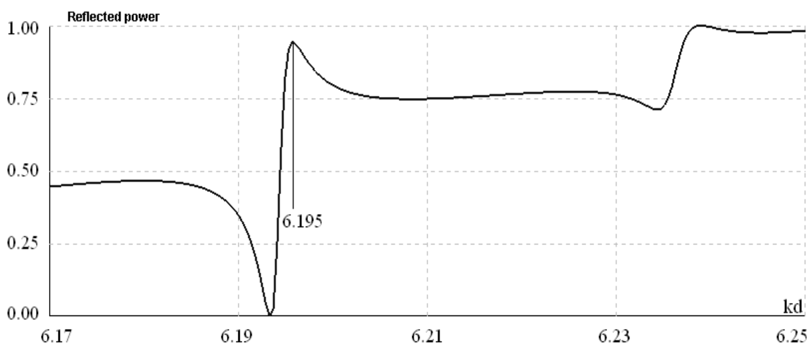


nax. 1.3.42 Sori velis diagrama

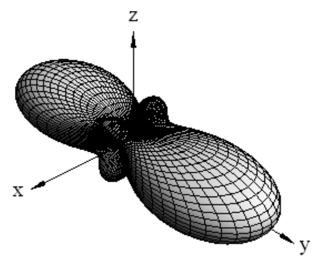


nax. 1.3.43 axlo velis ganawileba

meore SemTxvevaSi, rodesac *kd*=2.62, diagramaSi SeimCneva bevri foToli radgan es ufro maRali rigis rezonansia.

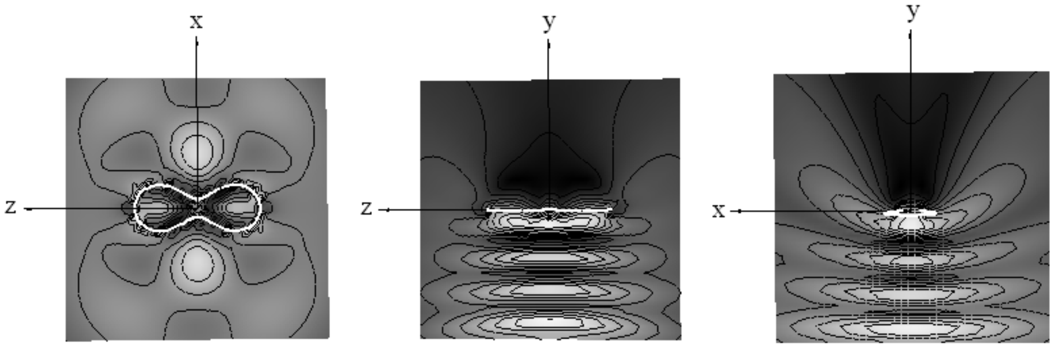


nax. 1.3.44 gadasxivebuli simZlavris damokidebuleba *kd* sidideze



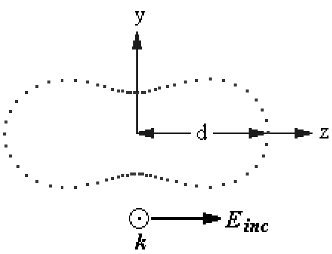
nax. 1.3.45 Sori velis diagrama

Semdeg ganxilul iqna SemTxveva, rodesac dacemuli velis wyaro imyofeba (0, -3*d*, 0) wertilSi da gaaCnia *Z* polarizacia. qveviT moyvanilia danormirebuli gadasxivebuli simZlavris damokidebuleba *kd* sidideze (nax. 1.3.44). gamoTvlebi gviCvenebs, rom *X* polarizaciis SemTxvevisagan gansxvavebiT, aq gadasxivebuli simZlavre erTi rigiT ufro maRali gamodis. rogorc moyvanili grafiki gviCvenebs, SemTxveva kd=6.195 Seesabameba rezonanss. qveviT moyvanilia Soris velis diagrama (nax. 1.3.45) da axlo velis ganawileba (nax. 1.3.46) am rezonansis dros. aqac gamokveTilia energiis gavrcelebis ori mimarTuleba - OY RerZis gaswvriv da mis sawinaaRmdegod, Tumca X polarizaciisagan gansxvavebiT es energia metad ufro mimarTulia.

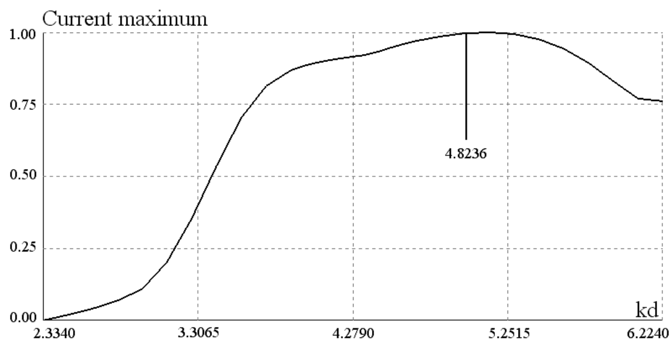


nax. 1.3.46 axlo velis ganawileba

ganieri rezonansis SemTxveva. Tu SevcvliT kasinis elementis formas, maSin SegviZlia miviRoT ufro ganieri rezonansi. naxazze 1.3.47 moyvanilia elementis geometria, rodesac mis parametrebs Soris arsebobs damokidebuleba $(a = 1.1c)$. dacemuli velis wyaro imyofeba wertilSi $(4d, 0, 0)$.

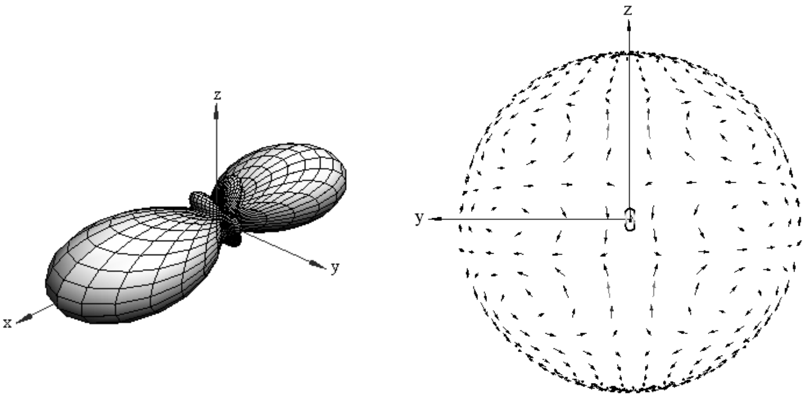


nax. 1.3.47 elementis geometria

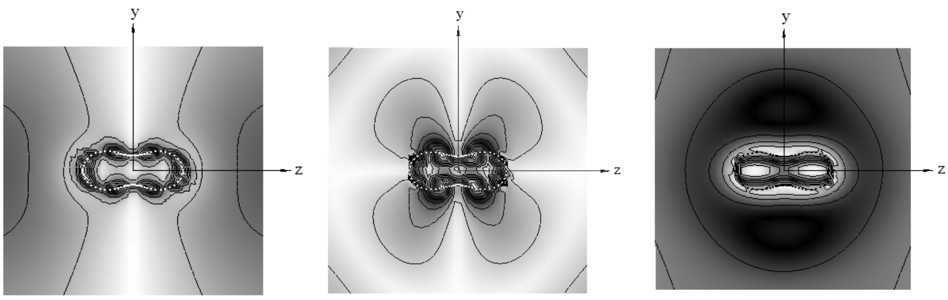


nax. 1.3.48 denis maqsimumis damokidebuleba kd parametrze

dacemuli veli vrceldeba OX RerZis sawinaaRmdego mimarTulebiT da gaaCnia OZ polarizacia. areSi $kd = 2.3340 - 6.2240$, elements gaaCnia ganieri rezonansi (nax. 1.3.48). Semdeg naxazze moyvanilia Soris velis diagrama rezonansis dros (nax. 1.3.49 a)). am SemTxvevaSi vels mbrunavi polarizacia gaaCnia (nax. 1.3.49 b)) rac ufro ukeTesad Cans animaciisas. gabneul vels gaaCNia ori didi foToli. axlo velis ganawileba rezonansis dros moyvanilia naxazze 1.3.50.

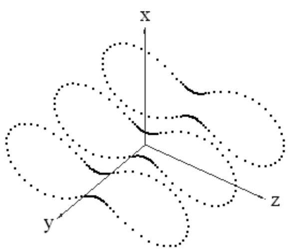


nax. 1.3.49. a) Sori velis diagrama (kd=4.8236), b) mbrunavi polarizacia

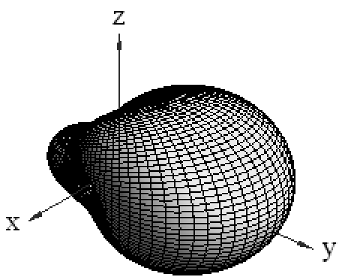


nax. 1.3.50 axlo velis komponentebis ganawileba

sami elementis SemTxveva rodesac dacemuli velis wyaro maTTan axlos imyofeba. ganixileba sami kasini Tavisufal sivrceSi (nax. 1.3.51). dacemuli talRas X polarizacia gaaCnia da misi wyaro imyofeba wertilSi (0, -3d, 0).

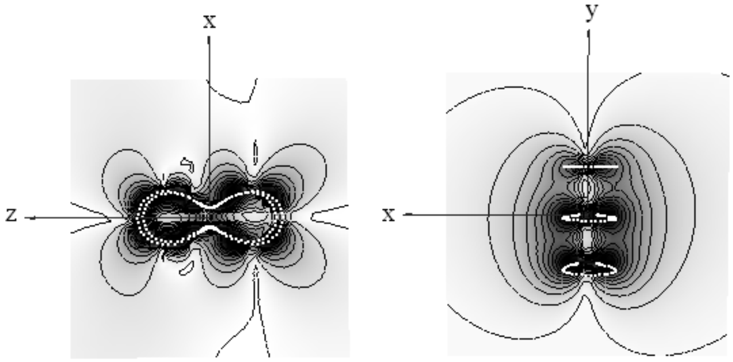


nax. 1.3.51 struqturis geometria

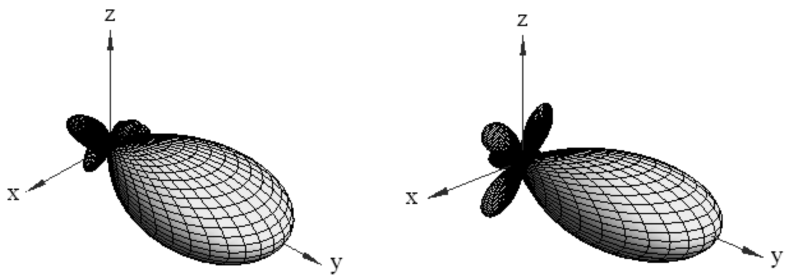


nax. 1.3.52 Sori velis diagrama

erTi kasinis rezonansul sixSireze (kd=1.79) napovnia rezonansuli manZili maT Soris, rodesac aqvs adgili ormag rezonanss (d_2=0.714d). naxazze 1.3.52, 1.3.53 moyvanilia Sori velis diagrama da axlo velis ganawileba. rogorc vxedavT, ormagi rezonansis dros gamosxivebul energias gaaCnia garkveuli mimarTuleba sivrceSi.

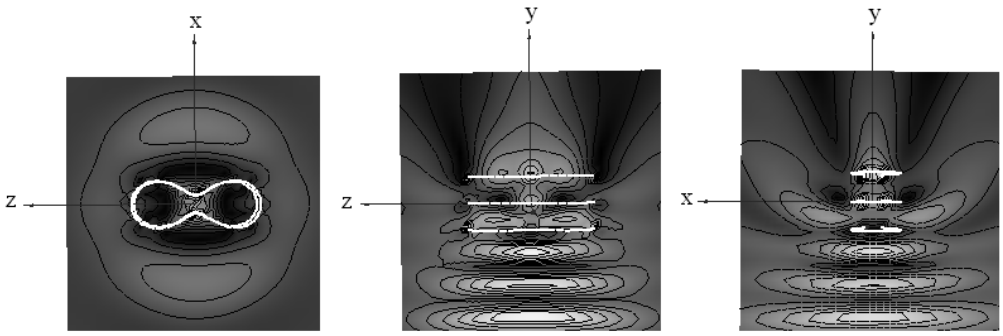


nax. 1.3.53 axlo velis ganawileba

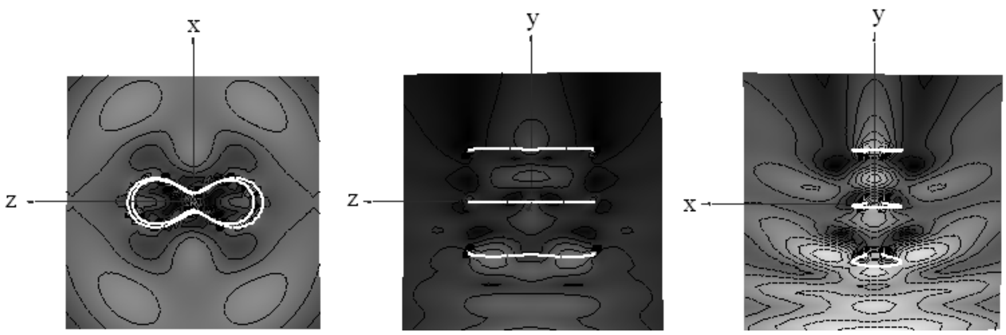


a) b)

nax. 1.3.54 Sori velis diagrama a) d_2=0.04d, b) d_2=0.84d



a)

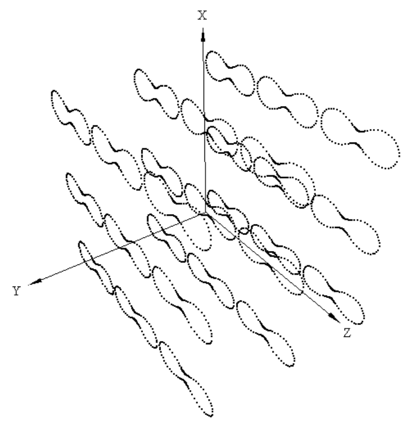


b)

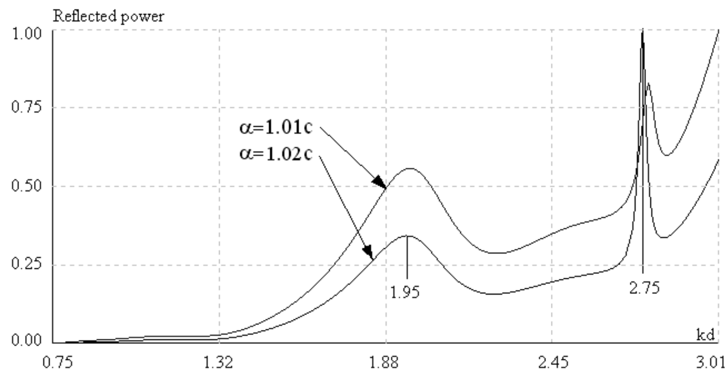
nax. 1.3.55 axlo velis ganawileba a) d_2=0.04d, b) d_2=0.84d

Z polarizaciis dros, rezonansul sixSireze (kd=6.195) kvlav napovnia optimaluri manZili elementebs Soris, rodesac aqvs adgili ormag rezonanss. am manZilis mniSvneloba d_2=0.04d da aseve d_2=0.84d. qveviT moyvanilia Sori velis diagramebi da axlo velis E_z mdgenelis ganawileba am ori rezonansis SemTxvevaSi (nax. 1.3.54 a), b) da nax. 1.3.55 a), b)). rogorc vxedavT, ormagi rezonansis dros gadasxivebuli veli ufro mimarTulia.

meseri Tavisufal sivrceSi. Semdeg ganxilul iqna kasinis elementebisagan Semdgari perioduli meseri (nax. 1.3.56), romelsac *OY* mimarTulebidan ecema brtyeli, *OX* da *OZ* polarizaciebis mqone talRa. erTi elementis rezonansul SemTxvevisaTvis (kd=4.36) iqna napovni optimaluri manZili ($d_1 = d_2 = d_3 = 2.09d$) elementebs Soris, rodesac aqvs adgili ormag rezonanss. naxazze 1.3.57 moyvanilia gadasxivebuli simZlavris damokidebuleba *kd* parametrze.

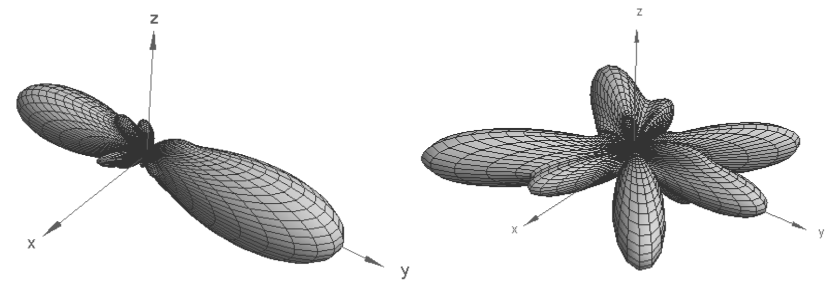


nax. 1.3.56 mesris geometria



nax. 1.3.57 gadasxivebuli simZlavris damokidebuleba *kd* parametrze

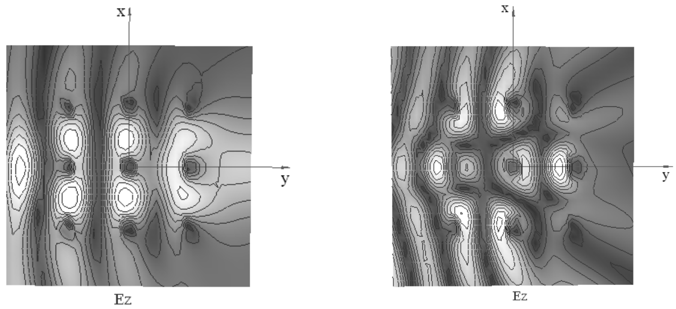
Cven vxedavT, rom wina SemTxvevisagan gansxvavebiT, kasinis *a* koeficientis cvlilebiT rezonansis wanacvleba ar xdeba. Semdeg moyvanilia Sori velis diagrama da aseve axlo velis E_z komponentis ganawileba rezonansebis SemTxvevaSi, rodesac $a = 1,02c$, $kd = 1.95$ da $kd = 2.75$ (nax. 1.3.58 da 1.3.59).



a) b)

nax. 1.3.58 Sori velis diagrama: a) $kd = 1.95$, b) $kd = 2.75$

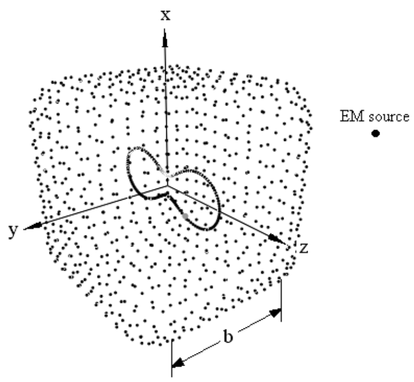
rezonansis dros, rodesac $kd = 1.95$, gamosxivebuli veli ZiriTadad *OY* RerZis gaswvriv vrceldeba. Semdegi rezonansis dros ($kd = 2.75$), *XOY* sibrtyeSi gamokveTilia gabneuli velis ramodenime mimarTuleba. gabneuli velis kvlevam gviCvena aseve, rom mas gaaCnia elifsuri polarizacia elifsurobis koeficientiT 1:2.



a) b)

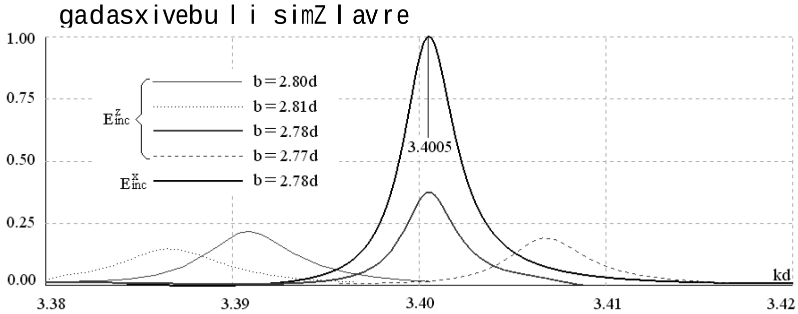
nax. 1.3.59 axlo velis ganawileba: a) $kd = 1.95$, b) $kd = 2.75$

elementi dieleqtrikis SigniT. ganxilul iqna SemTxveva rodesac erTi kasinis elementi moTavsebuli iyo $\varepsilon = 4$ SeRwevadobis mqone dieleqtrikuli kubis SigniT (nax. 1.3.60). dacemuli velis wertilovani wyaro imyofeba (0, -4*d*, 0) wertilSi.

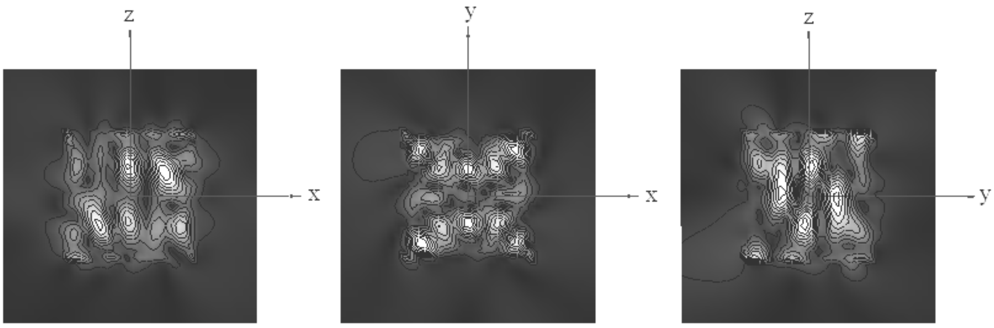


nax. 1.3.60 kasinis elementi dieleqtrikis SigniT

Semdeg naxazze (nax. 1.3.61) moyvanilia maqsimumze danormirebuli gadasxivebuli simZlavris damokidebuleba *kd* parametrze kubis sxvadasxva *b* zomis SemTxvevaSi. rogorc vxedavT, am simZlavris maqsimaluri mniSvneloba (rezonansi elementsa da dieleqtrikis Soris) miiRweva rodesac *kd*=3.4005, dacemuli velis *OX* polarizaciis SemTxvevaSi. axlo velis ganawileba am rezonansis SemTxvevaSi moyvanilia naxazze 1.3.62.

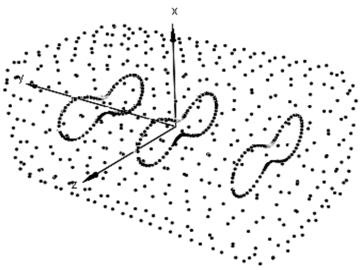


nax. 1.3.61 gadasxivebuli simZlavris damokidebuleba *kd* parametrze

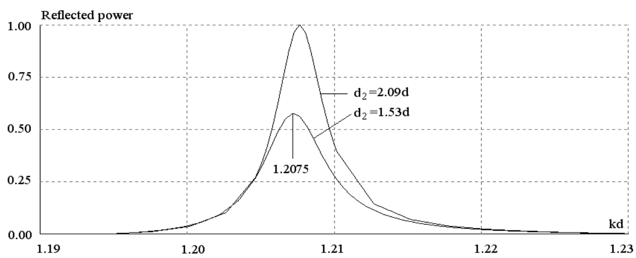


nax. 1.3.62 axlo velis ganawileba

meseri dieleqtrikis SigniT. aseve ganxilul iqna SemTxveva rodesac sami kasinis elementi moTavsebuli $\varepsilon = 4$ SeRwevadobis mqone dieleqtrikul paralelepipedSi (nax. 1.3.63). manZili elementebs Soris $1.53d$-s Seadgens. dieleqtrikis zomebia $2.37d \times 2.79d \times 4.87d$ da SemosazRvrulia gluvi zedapiriT. momrgvalebis radiusia $0.56d$. naxazze 1.3.64 moyvanilia danormirebuli maqsimalur mniSvnelobaze gadasxivebuli simZlavris damokidebuleba kd parametrze. aq moyvanilia ori mrudi, romlebic Seesabamebian sxvadasxva manZilebs elementebs Soris. rogorc vxedavT, gadasxivebuli simZlavre izrdeba rodesac manZili xdeba $2.09d$-s toli.

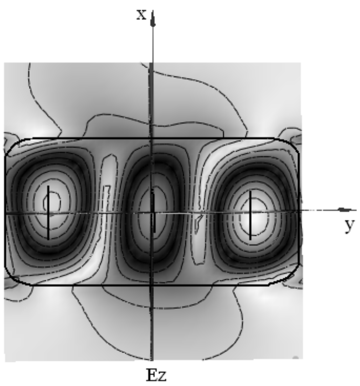


nax. 1.3.63 structuris geometria



nax. 1.3.64 gadasxivebuli simZlavris damokidebuleba kd parametrze

Semdeg naxazze (nax. 1.3.65) moyvanilia axlo velis ganawileba rezonansis SemTxvevaSi ($kd = 1.2075$). gabneuli velis analizi aCvenebs, rom mas gaaCnia mbrunavi polarizacia romelic axlosaa wriulTan.



nax. 1.3.65 axlo velis ganawileba

daskvna

gamokvleulia metalo-dieleqtrikuli struqturebis zogierTi eleqtrodinamikuri Tvisebebi. ganxilulia mesris elementis ramodenime gansxvavebuli forma. moyvanilia problemis gadaWris Teoriuli safuZvlebi. ZiriTad mizans warmoadgenda aseTi struqturebis Seswavla rezonansul sixSireebze, rodesac isini kompleqsuri garemos Tvisebebs amJRavneben. kerZod, miRebulia kiraluri da aseve uaryofiTi gardatexis mqone struqtura. ganxilulia axali saxis elementi - “kasini” romelsac gaaCnia farTo sixSirul diapazonSi rezonansuli Tvisebebi. ganxilulia aseve antenuri amocana, rodesac dacemuli velis wyaro struqturis SigniT imyofeba. ricxviTi gamoTvlebis saSualebiT SerCeulia struqturis optimaluri parametrebi brtyeli an mimarTuli diagramis misaRebad. gamoTvlebi Catarebulia damxmare gamomsxiveblebis meTodis gamoyenebiT. ricxviTi gamoTvlebis paralelurad mowmdeboda gamoTvlebis cdomileba romelic ar aRemateboda 2-5%.

unda aRiniSnos, rom am TavSi moyvanili zogierTi Sedegi statiis saxiT miRebul iqna dasabeWdad imfaqt-faqtorian jurnalSi “Journal of Communications Technology and Electronics” [43].

Tavi II

brtyeli eleqtromagnituri talRis difraqcia usasrulo orperiodul meserze Tavisufal garemoSi

zogadi mimoxilva

es Tavi exeba brtyeli talRis difraqcias usasrulo orperiodul meserze [44]. mesris elementi warmoadgens rezonansuli Tvisebebis, mcire eleqtruli radiusis da sasruli sigrZis mqone gamtars. es amocana dResdReobiT metad aqtualuria, radgan kompleqsuri masalebis Tvisebebis Seswavla swored periodul struqturebze difraqciis amocanis amoxsnaze daiyvaneba. praqtikaSi saqme gvaqvs sasruli zomis mesrebTan, magram Tu am mesris elementebis raodenoba didia, misi modelireba did kompiuterul resursebTan xdeba dakavSirebuli. amitom ufro martivia ganxiluli iqnas difraqciis amocana usasrulo periodul struqturaze, romelic maTematikurad ufro martivad amoixsneba da amave dros veli axlo areSi faqtiurad ar gansxvavdeba sasruli zomis mesris SemTxvevisgan.

perioduli struqturebis eleqtromagnituri Tvisebebis gamokvleva intensiurad daiwyo XX saukunis SuawlebSi. 1954 wels ueitma ganixila sasruli gamtareblobis mqone usasrulo sigrZis mavTulebisgan Semdgari meseri. faqtorizaciis meTodis gamoyenebiT vainSteinis mier ganxiluli iqna usasrulo sigrZis zolebisagan Semdgari meseri, romlis periodi zolis siganes udrida. Sestopalovis mier gamoyenebuli iqna riman - hilbertis ricxviTi meTodi difraqciuli amocanebis amosaxsnelad.

moyvanil SromebSi ganixileboda erTperioduli mesrebi, anu romlebic iyvnen periodulebi mxolod erTi mimarTulebiT

Tavi II Sedgeba 3 paragrafisgan:

pirveli paragrafis dasawyisSi gamoyvanilia puasonis ajamvis formula, romlis gamoyeneba amartivebs Semdgom gamoTvlebs. Semdeg xdeba ZiriTadi amocanis dasma da moyvanilia misi amoxsnis meTodi. naCvenebia, rom mesris mier gabneuli veli warmoadgens brtyeli talRebis superpozicias.

meore paragrafSi xdeba ucnobi denis amplitudebis gansazRvra mesris elementebSi. es amocana daiyvaneba wrfiv algebrul gantolebaTa sistemaze. moyvanilia aseve napovni gabneuli velis gamosaxuleba.

mesame paragrafSi moyvanilia kompiuteruli modelirebis saSualebiT miRebuli ricxviTi eqsperimentis Sedegebi.

$2.1 brtyeli talRis difraqcia usasrulo orperiodul struqturaze Tavisufal garemoSi

Semdgom gamoTvlebSi saWiro iqneba usasrulo mwkrivebTan muSaoba da amitom moxerxebulia maTi gardaqmna cnobili puasonis ajamvis formulis gamoyenebiT. es gardaqmna gulisxmobs mocemuli mwkrivis Canacvlebas sxva mwkriviT, romelic gacilebiT ufro swrafad ikribeba.

imisaTvis, rom miviRoT aRniSnuli formula, ganvixiloT Semdegi saxis funqcia:

$$\sum_{n=-\infty}^{+\infty} \delta(x-n),$$

sadac δ delta funqciaa. gasagebia, rom am funqciis mniSvneloba ar Seicvleba, Tu mis x arguments davumatebT raime naturalur ricxvs. es imas niSnavs, rom igi periodulia da misi periodi erTis tolia. amis gamo, igi SeiZleba warmodgenil iqnas furies mwkriviT:

$$\sum_{n=-\infty}^{+\infty} \delta(x-n) = \sum_{\alpha=-\infty}^{+\infty} a_\alpha e^{i2\pi\alpha x}.$$

ucnobi a_α furies koeficientebis sapovnelad gavamravloT tolobis orive mxare $e^{-i2\pi px}$, sadac p fiqsirebuli naturaluri ricxvia da gavaintegroT periodis fargl ebSi:

$$\sum_{n=-\infty}^{+\infty} \int_{-1/2}^{1/2} \delta(x-n) e^{-i2\pi px} dx = \sum_{\alpha=-\infty}^{+\infty} a_\alpha \int_{-1/2}^{1/2} e^{i2\pi(\alpha-p)x} dx.$$

marcxena mxareSi gagvaCnia mwkrivis mxolod nulovani wevri, romelic udris erTs. yvela danarCeni wevri nulis tolia, radgan $n \notin [-1/2, 1/2]$ rodesac $n \neq 0$ da Sesabamisad $\delta(x-n) = 0$. marjvena mxareSi

$$\int_{-1/2}^{1/2} e^{i2\pi(\alpha-p)x} dx = \frac{\sin(\pi(\alpha-p))}{\pi(\alpha-p)} = \begin{cases} 1, \alpha = p \\ 0, \alpha \neq p \end{cases}.$$

maSasadame $a_p = 1$ da furies yvela koeficienti erTis tolia. amitom

$$\sum_{n=-\infty}^{+\infty} \delta(x-n) = \sum_{\alpha=-\infty}^{+\infty} e^{i2\pi\alpha x}.$$

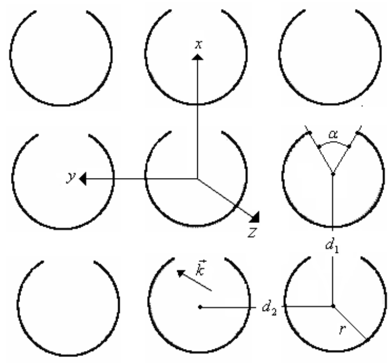
Tu gavamravlebT bolo tolobis orive mxares raime $f(x)$ funqciaze da gavaintegrebT intervalSi $(-\infty, +\infty)$, maSin sabolood miviRebT

$$\sum_{n=-\infty}^{+\infty} f(n) = \sum_{\alpha=-\infty}^{+\infty} \int_{-\infty}^{+\infty} f(x) e^{i2\pi\alpha x} dx.$$

es aris puasonis ajamvis formula. ormagi mwkrivis SemTxvevaSi SeiZleba am formulis orjer gamoyeneba, ris Sedegadac miviRebT formulas

$$\sum_{m=-\infty}^{+\infty} \sum_{n=-\infty}^{+\infty} \varphi(m,n) = \sum_{\alpha=-\infty}^{+\infty} \sum_{\beta=-\infty}^{+\infty} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \varphi(x,y) e^{i2\pi(\alpha x+\beta y)} dxdy.$$

amocanis dasma. ganvixiloT $z = 0$ sibrtyeSi usasrulo orperioduli struqtura, romelic Sedgeba rezonansuli Tvisebebis mqone gamtar elementebisagan. naxazze 2.1.1 moyvanilia meseri, romlis elementi Ria rgols warmoadgens, Tumca qvemoT Camoyalibebuli Teoria zogadia da is SeiZleba gamoyenebul iqnas sxva, ufro rTuli formis mqone elementebis SemTxvevaSic.



nax. 2.1.1. mesris geometria

igulisxmeba, rom elements gaaCnia mcire dr_0 radiusi. mesris periodebi OX da OY RerZebis gaswvriv Sesabamisad avRniSnoT rogorc d_1 da d_2. ganxilul struqturas ecema droSi harmoniuli brtyeli eleqtromagnituri talRa

$$\vec{E}_{inc}(\vec{r}) = \vec{E}_0 e^{i\vec{k}\vec{r}}, \quad \vec{H}_{inc}(\vec{r}) = \vec{H}_0 e^{-i\vec{k}\vec{r}},$$

$$H_0 = E_0/Z_0, \quad k = 2\pi/\lambda = \omega\sqrt{\varepsilon_0\mu_0}, \quad Z_0 = \sqrt{\mu_0/\varepsilon_0},$$

aq λ talRis sigrZea, Z_0 Tavisufali garemos talRuri winaRobaa, $\vec{k}$ talRuri veqtoria, $\vec{r}\{x,y,z\}$ dakvirvebis wertilis radiusveqtors warmoadgens. drois maxasiaTebelia $e^{-i\omega t}$.

dacemuli talRa aRZravs mesris yovel elementSi ucnobi amplitudebis mqone denebs romlebic gadaasxiveben meorad $\vec{E}(\vec{r})$, $\vec{H}(\vec{r})$ vels. Cveni amocanaa gavnsazRvroT ucnobi denebi da vipovoT mesris mier gabneuli $\vec{E}(\vec{r})$, $\vec{H}(\vec{r})$ veli.

usasrulo periodulobis da brtyeli dacemuli talRis gamo mesris elementebi msgavs pirobebSi imyofebian da yovel maTganze erTi amplitudis denebi aRiZvreba, romlebic mxolod fazebiT gansxvavdebian erTmaneTisagan. gansxvavebas am fazebs Soris ar eqneba adgili marTobuli dacemis SemTxvevaSi. aqedan gamomdinare, gadasxivebuli veli aseve periodulia da yovel $d_1 \times d_2$ areSi gaaCnia erTi da igive saxe.

gadasxivebuli velis periodulobis gamo, sakmarisia igi SeviswavloT mxolod erT romelime $d_1 \times d_2$ areSi da davakmayofiloT misTvis sasazRvro piroba mxolod am areSi myof elementze.

mesris yoveli elementi avRniSnoT rogorc l_{mn}. aq pirveli m indeqsi gviCvenebs elementis rigis nomers meserSi x koordinatis gaswvriv, xolo n indeqsi - svetis nomers y koordinatis gaswvriv. ($-\infty < m,n < +\infty$). Tu $\vec{r}_0\{x_0, y_0, 0\}$ da $\vec{r}_{mn}\{x_n, y_m, 0\}$ Sesabamisad mesris centraluri da l_{mn} elementis gaswvriv aRebuli radiusveqtoria, maSin

$$x_n = x_0 + nd_1, \quad y_m = y_0 + md_2.$$

amocanis amoxsnis meTodi. mesris yoveli l_{mn} elementis mier gadasxivebuli veli $\vec{E}_{mn}(\vec{r})$, $\vec{H}_{mn}(\vec{r})$ - iT aris aRniSnuli. sruli veli ganisazRvreba am velebis jamis saxiT:

$$\vec{E}(\vec{r}) = \sum_{m=-\infty}^{+\infty} \sum_{n=-\infty}^{+\infty} \vec{E}_{mn}(\vec{r}), \quad \vec{H}(\vec{r}) = \sum_{m=-\infty}^{+\infty} \sum_{n=-\infty}^{+\infty} \vec{H}_{mn}(\vec{r}).$$

rogorc iqna aRniSnuli, es veli perioduli da amitom sakmarisia misi gansazRvra erT romelime periodis fargleбSi. moxerxebulia arCeul iqnas is $d_1 \times d_2$ periodi, romelic mesris centralur elements Seicavs. ucnobi $\vec{E}(\vec{r})$, $\vec{H}(\vec{r})$ veli unda akmayofilebdes am elementis zedapirze Semdeg sasazRvro pirobas:

$$\vec{E}(\vec{r} + d\vec{r}_0) \cdot \vec{l}_0 = -\vec{E}_{inc}(\vec{r} + d\vec{r}_0) \cdot \vec{l}_0, \qquad (2.1.1)$$

sadac $\vec{l}_0$ warmoadgens elementis zedapiris gaswvriv aRebul tangencialur veqtors, $|d\vec{r}_0| = dr_0$ mesris elementis radiusia. sasazRvro piroba yvela danarCen elementze, periodulobis gamo, avtomaturad Sesruldeba.

TviToeul elementis mier gadasxivebuli veli veqtoruli da skalaruli potencialiT gamovsaxoT:

$$\vec{E}_{mn}(\vec{r}) = -grad\varphi_{mn}(\vec{r}) + i\omega\vec{A}_{mn}(\vec{r}), \quad \vec{H}_{mn}(\vec{r}) = (1/\mu_0)\, rot\vec{A}_{mn}(\vec{r}).$$

aqedan gamomdinare

$$\vec{E}(\vec{r}) = -grad\varphi(\vec{r}) + i\omega\vec{A}(\vec{r}), \quad \vec{H}(\vec{r}) = (1/\mu_0)\, rot\vec{A}(\vec{r}), \qquad (2.1.2)$$

sadac

$$\vec{A}(\vec{r}) = \sum_{m=-\infty}^{+\infty} \sum_{n=-\infty}^{+\infty} \vec{A}_{mn}(\vec{r}), \quad \varphi(\vec{r}) = \sum_{m=-\infty}^{+\infty} \sum_{n=-\infty}^{+\infty} \varphi_{mn}(\vec{r}) \ -$$

jamuri potencialebia.

dakvirvebis wertilSi $\vec{A}_{mn}(\vec{r})$ da $\varphi_{mn}(\vec{r})$ potencialebisTvis davwerT

$$\varphi_{mn}(\vec{r}) = (1/4\pi\varepsilon_0) \int_{l_{mn}} \sigma(l_{mn}) e^{ik|\vec{r}-\vec{r}_{mn}|} \Big/ |\vec{r} - \vec{r}_{mn}|\, dl, \quad \vec{A}_{mn}(\vec{r}) = (\mu_0/4\pi) \int_{l_{mn}} I(l_{mn}) e^{ik|\vec{r}-\vec{r}_{mn}|} \Big/ |\vec{r} - \vec{r}_{mn}|\, d\vec{l}.$$

$I(l_{mn})$ da $\sigma(l_{mn})$ warmoadgenen Sesabamisad l_{mn} elementSi aRZrul denis da muxtis ganawilebas. Tu $I(l)$ da $\sigma(l)$ warmoadgenen centraluri l elementis muxtis da denis ganawilebas, maSin

$$I(l_{nm}) = I(l) e^{i(nk_x d_1 + mk_y d_2)}, \quad \sigma(l_{mn}) = \sigma(l) e^{i(nk_x d_1 + mk_y d_2)}, \quad \sigma(l) = -(i/\omega)\, dI(l)/dl.$$

SevitanoT es formulebi sruli potencialebis gamosaxulebebSi, miviRebT:

$$\vec{A}(\vec{r}) = (\mu_0/4\pi) \int_l I(l) \sum_{m=-\infty}^{+\infty} \sum_{n=-\infty}^{+\infty} G(\vec{r}, \vec{r}_{mn})\, d\vec{l}, \qquad (2.1.3)$$

$$\varphi(\vec{r}) = (1/4\pi\varepsilon_0) \int_l \sigma(l) \sum_{m=-\infty}^{+\infty} \sum_{n=-\infty}^{+\infty} G(\vec{r}, \vec{r}_{mn})\, dl, \qquad (2.1.4)$$

sadac

$$G(\vec{r}, \vec{r}_{mn}) = e^{i(nk_x d_1 + mk_y d_2 + k|\vec{r}-\vec{r}_{mn}|)} \Big/ |\vec{r} - \vec{r}_{mn}|$$

grinis funqciaa.

veqtoruli da skalaruli potencialebis (2.1.3), (2.1.4) gamosaxulebebSi figurirebs grinis funqciebis ormagi mwkrivi

$$\sum_{m=-\infty}^{+\infty}\sum_{n=-\infty}^{+\infty} G(\vec r,\vec r_{mn}),$$

romlis krebadoba SegviZlia gacilebiT gavzardoT Tu gamoviyenebT puasonis miRebul formulas:

$$\sum_{m=-\infty}^{+\infty}\sum_{n=-\infty}^{+\infty} G(\vec r,\vec r_{mn}) = \sum_{q=-\infty}^{+\infty}\sum_{p=-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} G(\vec r,\vec r_{\vartheta\tau})e^{2i\pi(p\vartheta+q\tau)}d\vartheta d\tau .$$

am gardaqmnis Sedegad miviRebT

$$\sum_{m=-\infty}^{+\infty}\sum_{n=-\infty}^{+\infty} G(\vec r,\vec r_{mn}) = (2i\pi/d_1d_2)\sum_{q=-\infty}^{+\infty}\sum_{p=-\infty}^{+\infty}\gamma_{qp}(\vec r,\vec r_0), \qquad (2.1.5)$$

sadac

$$\gamma_{qp}(\vec r,\vec r_0) = e^{i\vec k_{qp}(\vec r-\vec r_0)}\Big/\sqrt{k^2-k_{p,x}^2-k_{q,y}^2},$$

$$\vec k_{qp} = \vec k_{qp}\{k_{p,x},k_{q,y},k_{qp,z}\},\ \ k_{p,x}=k_x+2\pi p/d_1,\ \ k_{q,y}=k_y+2\pi q/d_2,\ \ k_{qp,z}=\operatorname{sgn}(z)\sqrt{k^2-k_{p,x}^2-k_{q,y}^2}.$$

rogorc vxedavT, marjvena mxare warmoadgens brtyeli talRebis (harmonikebis) jams. am jamis is komponentebi, romelTaTvis sruldeba utoloba

$$k^2 > k_{p,x}^2 + k_{q,y}^2,$$

aramilevad speqtralur komponentebs warmoadgenen da es - dabali rigis, komponentebia. SedarebiT maRali rigis komponentebi, romliTaTvis sruldeba utoloba

$$k^2 < k_{p,x}^2 + k_{q,y}^2,$$

eksponencialurad milevadia da maTi mniSvneloba mesridan daSorebiT sakmaod swrafad mcirdeba.

SevitanoT (2.1.5) potencialebis (2.1.3), (2.1.4) gamosaxulebebSi da movaxdinoT integrireba jamis SigniT. Tu davubrundebiT m, n indeqsebs, maSin davwerT

$$\vec A(\vec r) = (i\mu_0/2d_1d_2)\sum_{m=-\infty}^{+\infty}\sum_{n=-\infty}^{+\infty}\int_l I(l)\gamma_{mn}(\vec r,\vec r_0)d\vec l, \qquad (2.1.6)$$

$$\varphi(\vec r) = (i/2\varepsilon_0d_1d_2)\sum_{m=-\infty}^{+\infty}\sum_{n=-\infty}^{+\infty}\int_l \sigma(l)\gamma_{mn}(\vec r,\vec r_0)dl, \qquad (2.1.7)$$

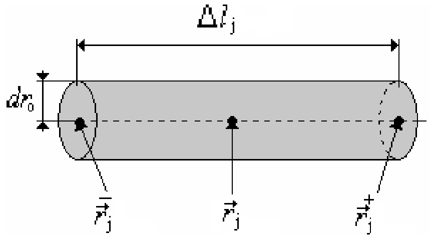
$$\gamma_{mn}(\vec r,\vec r_0) = e^{i\vec k_{mn}(\vec r-\vec r_0)}\Big/\sqrt{k^2-k_{n,x}^2-k_{m,y}^2},\ \ \vec k_{mn} = \vec k_{mn}\{k_{n,x},k_{m,y},k_{mn,z}\},$$

$$k_{n,x}=k_x+2\pi n/d_1,\ \ k_{m,y}=k_y+2\pi m/d_2,\ \ k_{mn,z}=\operatorname{sgn}(z)\sqrt{k^2-k_{n,x}^2-k_{m,y}^2}.$$

aq m, n indeqsebi ukve gviCveneben harmonikis nomers da ara struqturis elementis nomers, rogorc Tavidan iTvleboda. maSasadame Cveni amocanaa vipovoT denis da muxtis ganawileba centralur elementSi.

$2.2 gabneuli velis zogadi gamosaxuleba da ucnobi denebis gansazRvra

warmovidginoT centraluri elementi rogorc N raodenobis segmentebis erToblioba. yoveli aseTi segmenti davaxasiaToT Sua da kidura wertilebiT, xolo maTi radiusveqtorebi Sesabamisad $\vec r_j$, $\vec r_j^{\,+}$ da $\vec r_j^{\,-}$ -T avRniSnoT (nax. 2.2.1).



Nnax. 2.2.1. segmentis geometria mesris elementSi

segmentebis raodenoba aviRoT imdenad didi rom SegveZlos ugulebelvyoT $I(l)$ denis cvlileba yovel maTganis gaswvriv. Cven vTvliT rom yovel Δl_j segmentSi ($j=1,2,...,N$) gaedineba ucnobi I_j amplitudis deni. es SesaZloa mxolod im SemTxvevaSi Tu am segmentis boloebSi imyofeba ori urTierTsapirispiro niSnis muxti $+q_j$ da $-q_j$ sadac

$$q_j=-(i/\omega)I_j\,.$$

integralisTvis romelic figurirebs (2.1.6) veqtorul potencialis gamosaxulebaSi, davwerT:

$$\int_l I(l)\gamma_{mn}(\vec r,\vec r_0)\,d\vec l\approx\sum_{j=1}^{N}I_j\gamma_{mn}(\vec r,\vec r_j)\Delta\vec l_j\,,$$

sadac $\vec r_j$ warmoadgens Δl_j segmentis Sua wertilis, xolo

$$\gamma_{mn}(\vec r,\vec r_j)=e^{i\vec k_{mn}(\vec r-\vec r_j)}\Big/\sqrt{k^2-k_{n,x}^2-k_{m,y}^2}\,. \qquad (2.2.1)$$

mivaqcioT yuradReba imas, rom yoveli Δl_j segmenti qmnis or skalarul potencials romelic Seesabameba $+q_j$ da $-q_j$ muxtebs. amitom (2.1.7) gamosaxulebaSi mdebare integralisTvis davwerT:

$$\int_l \sigma(l)\gamma_{mn}(\vec r,\vec r_0)\,dl\approx-(i/\omega)\sum_{j=1}^{N}I_j\Delta\left[\gamma_{mn}(\vec r,\vec r_j)\right],$$

sadac

$$\Delta\left[\gamma_{mn}(\vec r,\vec r_j)\right]=\gamma_{mn}\left(\vec r,\vec r_j^{\,+}\right)-\gamma_{mn}\left(\vec r,\vec r_j^{\,-}\right).$$

Cven ar vcvliT $\Delta\left[\gamma_{mn}(\vec r,\vec r_j)\right]$ sasrul sxvaobas diferencialiT radgan es iqneboda samarTliani gacilebiT ufro did N- sTvis.

Tu SevitanT integralebis am gamosaxulebebs (2.1.6) da (2.1.7) formulebSi, maSin gveqneba

$$\vec A(\vec r)=(i\mu_0/2d_1d_2)\sum_{j=1}^{N}I_j\sum_{m=-\infty}^{+\infty}\sum_{n=-\infty}^{+\infty}\gamma_{mn}(\vec r,\vec r_j)\Delta\vec l_j\,, \qquad (2.2.2)$$

$$\varphi(\vec r)=-(1/2\omega\varepsilon_0d_1d_2)\sum_{j=1}^{N}I_j\sum_{m=-\infty}^{+\infty}\sum_{n=-\infty}^{+\infty}\Delta\left[\gamma_{mn}(\vec r,\vec r_j)\right]. \qquad (2.2.3)$$

gabneuli velis gamosaxuleba. gabneuli velis gamosaxuleba SegviZlia vipovoT (2.1.2) formulebis saSualebiT:

$$\vec{E}(\vec{r}) = -grad\varphi(\vec{r}) + i\omega\vec{A}(\vec{r}), \quad \vec{H}(\vec{r}) = (1/\mu_0)\, rot\vec{A}(\vec{r}).$$

saidanac miviRebT

$$\vec{E}(\vec{r}) = (1/2\omega\varepsilon_0 d_1 d_2)\sum_{j=1}^{N} I_j \sum_{m=-\infty}^{+\infty}\sum_{n=-\infty}^{+\infty}\left\{k^2\gamma_{mn}(\vec{r},\vec{r}_j)\Delta\vec{l}_j + i\Delta\left[\gamma_{mn}(\vec{r},\vec{r}_j)\right]\vec{k}_{mn,z}\right\}, \qquad (2.2.4)$$

$$\vec{H}(\vec{r}) = (1/2d_1 d_2)\sum_{j=1}^{N} I_j \sum_{n=-\infty}^{+\infty}\sum_{m=-\infty}^{+\infty}\gamma_{mn}(\vec{r},\vec{r}_j)\left(\Delta\vec{l}_j \times \vec{k}_{mn}\right). \qquad (2.2.5)$$

davuSvaT axla, rom segmentebis N raodenoba sakmarisad didia imisaTvis, rom SegveZlos sasruli sxvaoba $\Delta\left[\gamma_{mn}(\vec{r},\vec{r}_j)\right]$ diferencialiT CavanacvloT. am SemTxvevaSi (2.2.4) da (2.2.5) formulebis analogiurad, davwerT

$$\vec{E}(\vec{r}) = (1/2\omega\varepsilon_0 d_1 d_2)\sum_{j=1}^{N} I_j \sum_{m=-\infty}^{+\infty}\sum_{n=-\infty}^{+\infty} e^{i\vec{k}_{mn}(\vec{r}-\vec{r}_j)}\left(k^2 - k_{n,x}^2 - k_{m,y}^2\right)^{-1/2}\left(\vec{k}_{mn}\times\left(\vec{k}_{mn}\times d\vec{l}_j\right)\right), \qquad (2.2.6)$$

$$\vec{H}(\vec{r}) = (1/2d_1 d_2)\sum_{j=1}^{N} I_j \sum_{n=-\infty}^{+\infty}\sum_{m=-\infty}^{+\infty} e^{i\vec{k}_{mn}(\vec{r}-\vec{r}_j)}\left(k^2 - k_{n,x}^2 - k_{m,y}^2\right)^{-1/2}\left(d\vec{l}_j \times \vec{k}_{mn}\right). \qquad (2.2.7)$$

ganvixiloT am sammagi jamis romelime erTi wevri (erTerTi harmonika):

$$\vec{E}_{j,mn}(\vec{r}) = (1/2\omega\varepsilon_0 d_1 d_2) I_j e^{i\vec{k}_{mn}(\vec{r}-\vec{r}_j)}\left(k^2 - k_{n,x}^2 - k_{m,y}^2\right)^{-1/2}\left(\vec{k}_{mn}\times\left(\vec{k}_{mn}\times d\vec{l}_j\right)\right),$$

$$\vec{H}_{j,mn}(\vec{r}) = (1/2d_1 d_2) I_j e^{i\vec{k}_{mn}(\vec{r}-\vec{r}_j)}\left(k^2 - k_{n,x}^2 - k_{m,y}^2\right)^{-1/2}\left(d\vec{l}_j \times \vec{k}_{mn}\right).$$

SeiZleba naCveneb iqnas, rom am veqtorebis modulebis Sefardeba sivrcis talRur winaRobas udris, rac warmoadgens brtyeli talRis erTerT Tvisebas. marTlac, maTi modulebi tolia:

$$E_{j,mn}(\vec{r}) = (1/2\omega\varepsilon_0 d_1 d_2) I_j e^{i\vec{k}_{mn}(\vec{r}-\vec{r}_j)}\left(k^2 - k_{n,x}^2 - k_{m,y}^2\right)^{-1/2} k\sqrt{k^2 dl^2 - \left(\vec{k}_{mn}\cdot d\vec{l}_j\right)^2},$$

$$H_{j,mn}(\vec{r}) = (1/2d_1 d_2) I_j e^{i\vec{k}_{mn}(\vec{r}-\vec{r}_j)}\left(k^2 - k_{n,x}^2 - k_{m,y}^2\right)^{-1/2}\sqrt{k^2 dl^2 - \left(\vec{k}_{mn}\cdot d\vec{l}_j\right)^2},$$

saidanac

$$E_{j,mn}(\vec{r})/H_{j,mn}(\vec{r}) = k/\omega\varepsilon_0 = \sqrt{\mu_0/\varepsilon_0} = Z_0, \quad H_{j,mn}(\vec{r}) = E_{j,mn}(\vec{r})/Z_0.$$

maSasadame, mesris mier gabneuli veli gamoisaxeba (2.2.6) da (2.2.7) formulebiT. es veli warmoadgens milevadi da aramilevadi brtyeli talRebis superpozicias. Cveni amocana dayvanilia imaze rom vipovoT denis I_i amplitudebi yovel segmentSi.

denis amplitudebis gansazRvra. denis ucnob amplitudebs unda gaaCndeT iseTi mniSvnelobebi rom sruldebodes sasazRvro piroba yoveli segmentis zedapirze:

$$\vec{E}(\vec{r}_\vartheta + d\vec{r}_0)\cdot d\vec{l}_\vartheta = -\vec{E}_{inc}(\vec{r}_\vartheta + d\vec{r}_0)\cdot d\vec{l}_\vartheta, \quad \vartheta = 1,2,\ldots,N. \qquad (2.2.8)$$

(2.2.6) da aseve dacemuli velis formulebis Tanaxmad

$$\vec{E}\left(\vec{r}_{\vartheta}+d\vec{r}_0\right)=\left(1/2\omega\varepsilon_0 d_1 d_2\right)\sum_{j=1}^{N} I_j \sum_{m=-\infty}^{+\infty}\sum_{n=-\infty}^{+\infty}\gamma_{mn}\left(\vec{r}_{\vartheta}+d\vec{r}_0,\vec{r}_j\right)\left(\vec{k}_{mn}\times\left(\vec{k}_{mn}\times d\vec{l}_j\right)\right),$$

$$\vec{E}_{inc}\left(\vec{r}_{\vartheta}+d\vec{r}_0\right)=\vec{E}_0 e^{i\vec{k}\left(\vec{r}_{\vartheta}+d\vec{r}_0\right)},$$

rac sasazRvro pirobaSi Casmis Sedegad mogvcems ucnobi I_j amplitudebis mimarT wrfiv algebrul gantolebaTa sistemas

$$\sum_{j=1}^{N} Z_{j\vartheta} I_j = 2\omega\varepsilon_0 d_1 d_2\left(\vec{E}_0\cdot d\vec{l}_{\vartheta}\right),\quad \vartheta=1,2,\ldots,N,$$

sadac

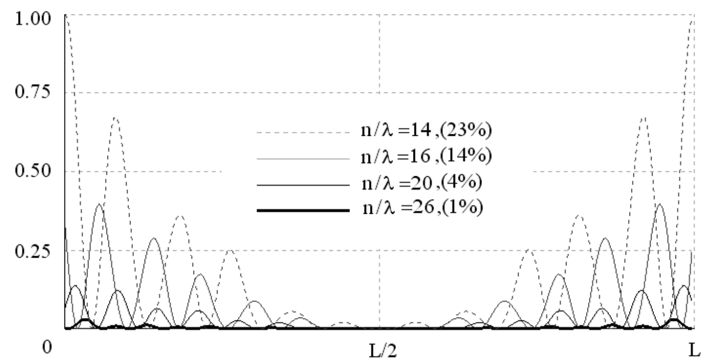
$$Z_{j\vartheta}=\sum_{m=-\infty}^{+\infty}\sum_{n=-\infty}^{+\infty} e^{i\left(\left(\vec{k}_{mn}-\vec{k}\right)\left(\vec{r}_{\vartheta}+d\vec{r}_0\right)-\vec{k}_{mn}\vec{r}_j\right)}\left(k^2-k_{n,x}^2-k_{m,y}^2\right)^{-1/2}\left(\vec{k}_{mn}\times\left(\vec{k}_{mn}\times d\vec{l}_j\right)\right)\cdot d\vec{l}_{\vartheta}.$$

am sistemis amoxsnis Sedegad vpoulobT denebis amplitudebs da (2.2.6), (2.2.7) formulebis saSualebiT gabneul vels.

$2.3 ricxviTi eqsperimentebis Sedegebi

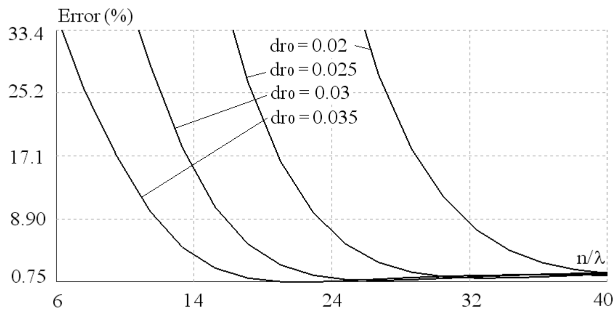
gamoTvlebis sizustis dadgena. imisaTvis, rom davrwmundeT miRebuli Sedegebis samarTlianobaSi, mniSvnelovania SerCeul iqnas mesris elementebis damxmare parametrebi: mcire segmentebis optimaluri sigrZe da mavTulebis radiusi dacemuli talRis sigrZesTan SedarebiT ($\Delta l_j/\lambda$, dr_0/λ). sxvanairad rom vTqvaT, unda davrwmundeT, rom Cveni gamoTvlebi samarTliania da miRebuli Sedegebis sizuste kontrolirebadia. Tu gaviTvaliswinebT, rom ganxiluli velebi akmayofileben talRur gantolebas da amasTanave puasonis gardaqmna aris samarTliani, maSin amoxsnis sizuste unda iyos damokidebuli sasazRvro pirobebis Sesrulebaze kolokaciis wertilebs Soris. am optimaluri parametrebis codna iZleva saSualebas maRali sizustiT gamovikvlioT usasrulo perioduli struqturebis eleqtrodinamikuri Tvisebebi.

naxazze 2.3.1 moyvanilia sasazRvro pirobis Sesrulebis damokidebuleba mavTulze, kolokaciis wertilebis sxvadasxva raodenobis SemTxvevaSi talRis sigrZis gaswvriv.



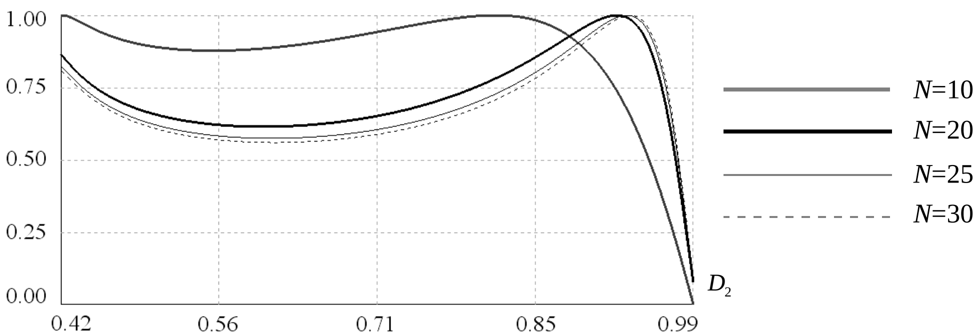
nax. 2.3.1 amoxsnis sizustis Semowmeba

moyvanili gadaxra sasazRvro pirobidan danormirebulia maqsimumze, romelic miiReba, rodesac gagvaCnia talRis sigrZeze mosuli mxolod 14 kolokaciis wertili. rogorc es suraTi gviCvenebs, 20 wertilis SemTxveva, anu rodesac $n/\lambda = 20$, sakmarisia imisaTvis, rom miviRoT samarTliani ricxviTi Sedegebi mesris eleqtrodinamikuri Tvisebebis gamosakvlevad. optimaluri damxmare parametrebis Seswavlam gviCvena rom sauleTeso miaxloveba eleqtrulad wvril mavTulTan miiRweva rodesac misi dr_0 raiusi imyofeba 0.02λ da 0.035λ - s farglebSi (nax. 2.3.2).



nax. 2.3.2 cdomilebis damokidebuleba kolokaciis wertilebis raodenobaze talRis sigrZis gaswvriv, mavTulis sxvadasxva radiusebisaTvis

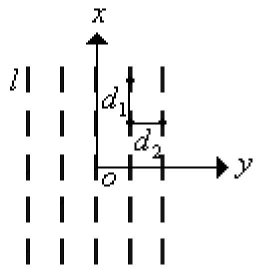
naxazze 2.3.3 moyvanili mrudebi warmoadgenen arekvlis koeficientis damikidebulebas mesris $D_2 = d_2/\lambda$ periodze sxvadasxva $N = n/\lambda$ kolokaciis wertilebis raodenobisaTvis Ria rgolebis SemTxvevaSi (nax. 2.1.1). rogorc Cans mrudi mniSvnelovnad aRar icvleba N=20 mniSvnelobidan.



nax. 2.3.3 arekvlis R koeficientis damokidebuleba mesris periodze D_1=0.5, R_0/λ=0.2, (N=10, 20, 25, 30).

Semdeg moyvanili Sedegebi miRebulia maTi sizustis winaswari SemowmebiT da fizikuri Sinaarsis gaanalizebiT. gamokvleuli struqturebis parametrebi moyvanilia uganzomilebo dayvanil erTeulebSi ($D_1 = d_1/\lambda$, $D_2 = d_2/\lambda$ da $L = l/\lambda$, sadac l- mesris elementis sruli sigrZea). amitom yvela es Sedegi rCeba samarTliani farTo sixSirul diapazonSi, sadac SeiZleba gamoyenebul iqnas maqsvelis klasikuri Teoria.

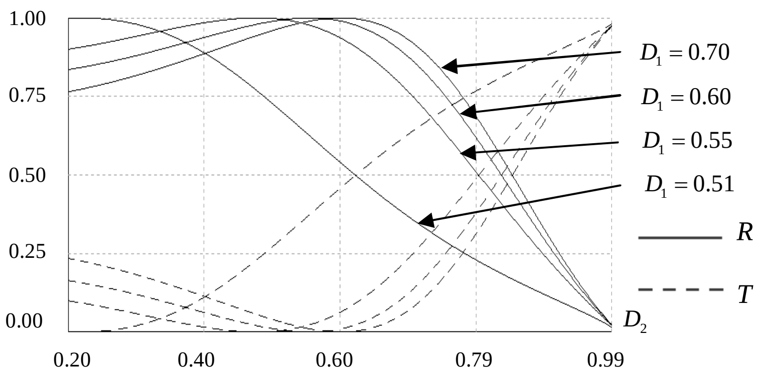
swor gamtarebisgan Semdgari usasrulo perioduli meseri. pirvel SemTxvevaSi ganxilul iqna mesris elementis umartivesi SemTxveva. elementi warmoadgens eleqtrulad mcire l sigrZis gamtars (nax. 2.3.4).



nax. 2.3.4 mesris geometria

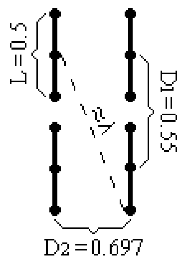
$$\vec{r}_{pq} = \vec{r}_{pq}\left\{(l/2)\vartheta + pd_1, qd_2, 0\right\},\ -1 \le \vartheta \le 1,\ -\infty < p < +\infty,\ -\infty < q < +\infty .$$

mesers marTobulad ecema erTeulovani amplitudes mqone brtyeli eleqtromagnituri talRa, romelic vrceldeba OZ RerZis sawinaaRmdego mimarTulebiT. naxazi 2.3.5 gviCvenebs R arekvlis da T gasvlis koeficientis damokidebulebas $D_2 = d_2/\lambda$ periodze sxvadasxva fiqsirebul $D_1 = d_1/\lambda$ periodis SemTxvevaSi. mesris elementis sigrZe aris rezonansi ($L = l/\lambda = 0.5$). energiis Senaxvis kanoni moiTxovs, rom arekvlis da gasvlis koeficientebis jami iyos erTis toli: $R+T=1$. rogorc Cans es piroba maRali sizustiT sruldeba, rac aseve miuTiTebs miRebuli Sedegis samarTlianobaze.

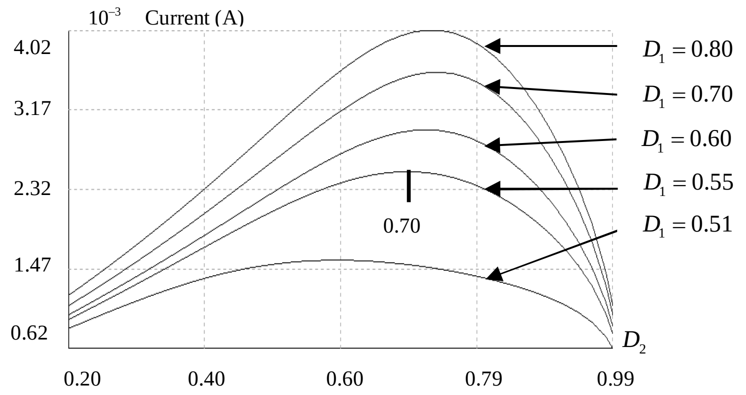


nax. 2.3.5 R arekvlis da T gasvlis koeficientebis damokidebuleba mesris periodebze

rodesac $D_1 = 0.51$ elementebi TiTqmis exebian erTmaneTs OX RerZis gaswvriv. rogorc vxedavT R=1 da T=0 rodesac $D_1 = 0.51$ da $D_2 = 0.2$ rac niSnavs imas, rom meseri iqceva rogorc amrekli zedapiri. D_1 periodis danarCen ganxilul mniSvnelobebisaTvis kvlav arsebobs iseTi D_2 periodi, rodesac meseri srulad ireklavs dacemul talRas. es movlena Seswavlil iqna siRrmiseulad, radgan igi dakavSirebulia rezonansul efeqtebTan. periodebis am mniSvnelobebze, zogierTi manZili mesris elementebs Soris dacemuli talRis λ sifrZis jeradi xdeba, rac rezonansul manZils Seesabameba (nax. 2.3.6).

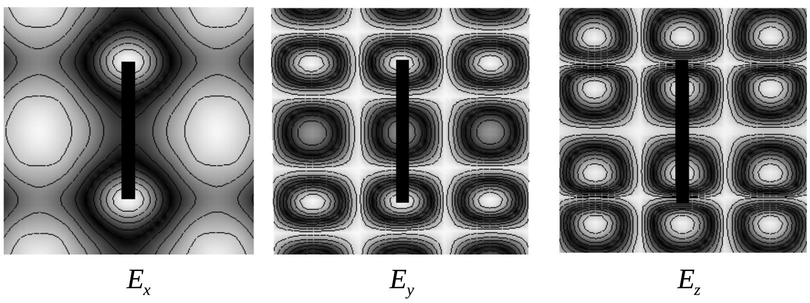


nax. 2.3.6 rezonansuli manZili elementebs Soris

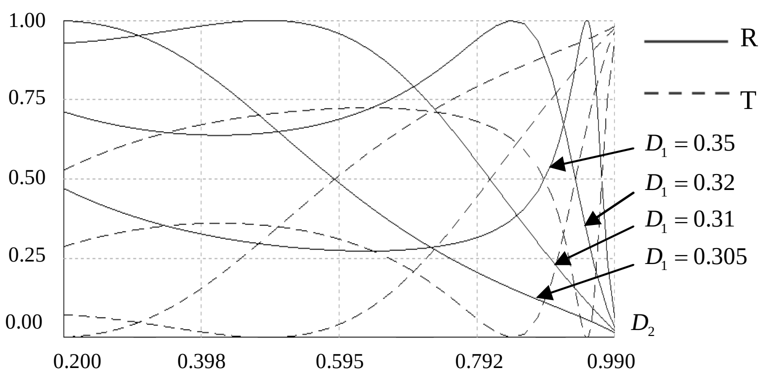


nax. 2.3.7 denis maqsimumis damokidebuleba mesris periodze. elementis sigrZea 0.5λ

2.3.5 naxazis analogiurad, interess warmoadgens aseve is, Tu rogoraa damokidebuli elementSi aRZruli denis maqsimumi mesris periodebze (nax. 2.3.7). yoveli denis mrudis piki naxazze 2.3.7 aseve Seesabameba rezonanss magram D_2 periodis rezonansuli mniSvnelobebi naxazebze 2.3.5 da 2.3.7 ar emTxvevian erTmaneTs. aRniSnuli ardamTxveva axsnil iqna Sesabamisi axlo velis Seswavlis Sedegad. denis rezonansis dros mniSvnelovnad Zlierdeba axlo veli, maSin rodesac Sor zonaSi, sadac gagvaCnia mxolod erTi aramilevadi speqtraluri komponenti, veli mniSvnelovnad ar icvleba.



nax. 2.3.8 axlo velis komponentebis ganawileba

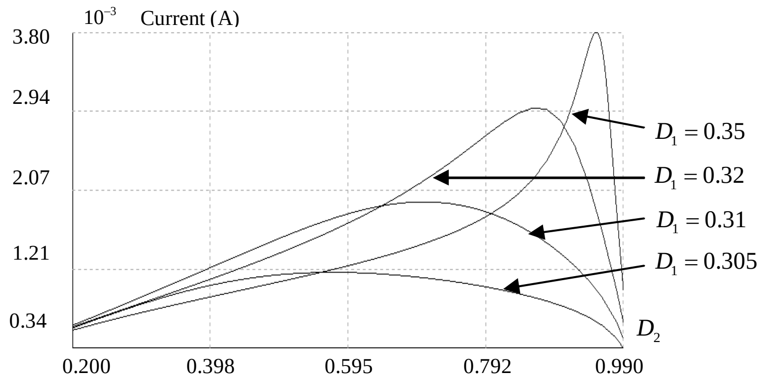


nax. 2.3.9 arekvlis da gasvlis koeficientis damokidebuleba mesris periodze. elementis sigrZea 0.3λ

naxazze 2.3.8 moyvanilia axlo velis komponentebis ganawileba $z=0$ sibrtyeSi erTerT rezonansis SemTxvevaSi rodesac $D_1=0.55$, $D_2=0.70$ da meseri iqceva rogorc sruliad amrekli zedapiri. am suraTze SegviZlia davinaxoT mdgari talRa mesris gaswvriv, romelic aniWebs mesers amrekl Tvisebebs. cxadia, rom aseTi Tviseba SeiZleba gamoyenebul iqnas praqtikaSi.

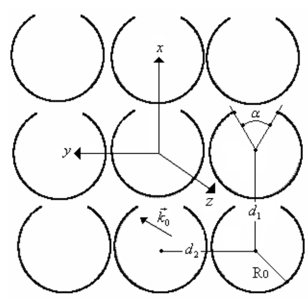
Semdeg ganxilul iqna SemTxveva rodesac mesris elementis sigrZe araa rezonansuli ($L=0.3$). naxazze 2.3.9 moyvanilia arekvlis da gasvlis koeficientis damokidebulebas D_2 periodze sxvadasxva fiqsirebul D_1 periodis SemTxvevaSi. rogorc vxedavT kvlav arsebobs am periodebis iseTi mniSvnelobebi, rodesac zogierTi manZili mezobel elementebs Soris xdeba rezonansuli da meseri kvkav iqceva rogorc amrekli zedapiri.

naxazze 2.3.10 moyvanilia analogiuri damokidebuleba elementebSi aRZruli denis maqsimumisaTvis. am mrudebis pikebi Seesabamebian rezonanss.

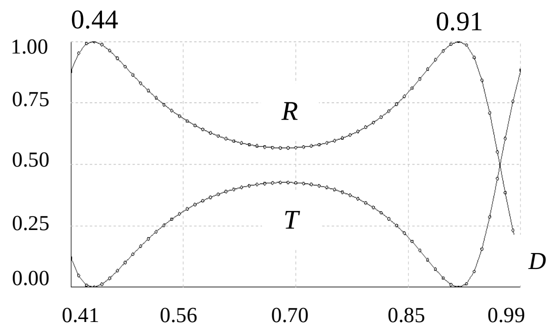


nax. 2.3.10 denis maqsimumis damokidebuleba mesris periodze. elementis sigrZea 0.3λ

Ria gamtari rgolebisgan Semdgari usasrulo perioduli meseri. Semdeg ganxilul iqna toli $D_1=D_2=D$ periodebis mqone usasrulo meseri, rodesac misi elementi Ria gamtar rgols warmoadgens (nax. 2.3.11). mesers ecema *X* polarizaciis mqone brtyeli talRa.

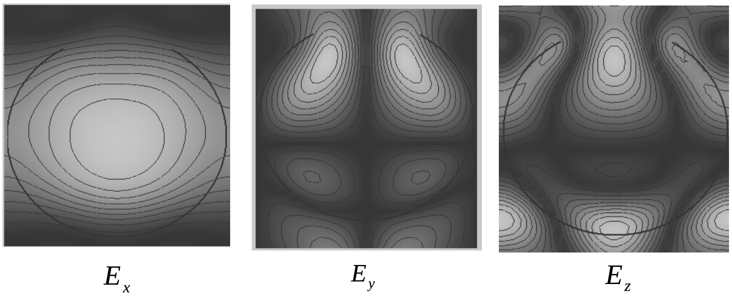


nax. 2.3.11 mesris geometria



nax. 2.3.12 arekvlis da gasvlis koeficientis damokidebuleba mesris periodze. $R_0/\lambda = 0.2$, N=20

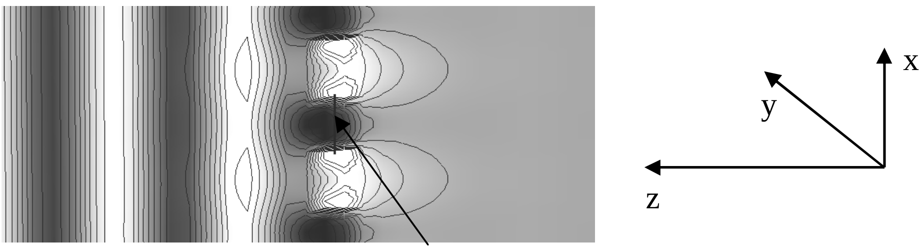
naxazze 2.3.12 moyvanilia R arekvlis da T gasvlis koeficientis damokidebuleba mesris D periodze. rgolis radiusia $R_0 = 0.2\ \lambda$, Ria seqtoris kuTxea 30^0, xolo kolokaciis wertilebis raodenoba rgolis gaswvriv aris $N = 20$. rogorc vxedavT, rodesac $D = 0.44$ da $D = 0.91$, adgili aqvs dacemuli talRis srul arekvlas. aseve, rogorc es iyo swori gamtarebis SemTxvevaSi, es sruli arekvla Seesabameba ormagi rezonansis SemTxvevas, anu rezonass mesris elementebs Soris. piroba $R + T = 1$ aseve sruldeba.



nax. 2.3.13 axlo velis komponentebis ganawileba

naxazze 2.3.13 moyvanilia axlo velis komponentebis ganawileba im SemTxvevaSi rodesac $R_0/\lambda = 0.24$ da mesris periodebi ar emTxveva erTmaneTs ($D_1 = 0.62$, $D_2 = 0.54$). velis daxatvis sibrtye wanacvlebulia mesris sibrtyidan, romelSic mas singularoba gaaCnia.

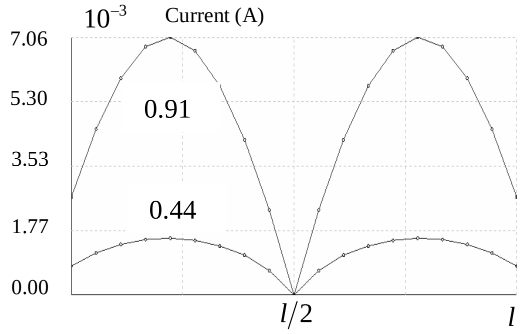
Semdeg kvlav ganixileba toli periodebis SemTxveva. naxazi 2.3.14 gviCvenebs axlo velis ganawilebas mesris perpendikularul sibrtyeSi rezonansuli periodis SemTxvevaSi $D = 0.91$ (nax. 2.3.12). suraTis marjvena mxareSi Cven vxedavT mdgar talRas rac niSnavs dacemuli talRis srul arekvlas. marjvena nawilSi dacemuli veli nawilobriv aRwevs, magram ar vrceldeba masSi, radgan arsebobs fazaTa sxvaoba eleqtrul da magnitur velს Soris amareSi.



elementi marTobul sibrtyeSi

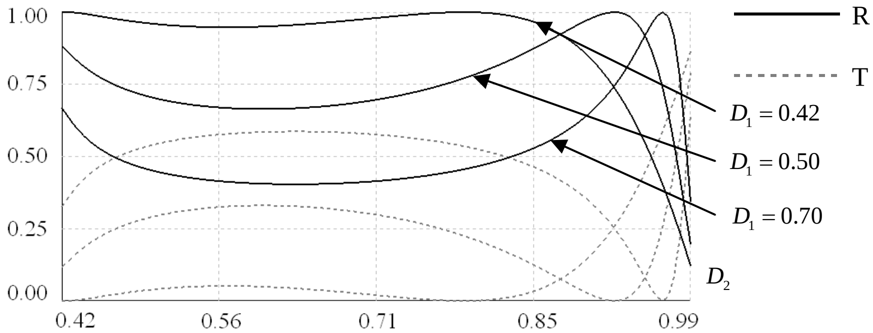
nax. 2.3.14 axlo velis ganawileba ormagi rezonansis SemTxvevaSi

Semdegi suraTi (nax. 2.3.15) gviCvenebs elementSi aRZruli denis ganawilebas rezonansis dros ($D = 0.44$ da $D = 0.91$, nax. 2.3.12). rogorc Cans, meore SemTxvevaSi gacilebiT ufro maRali amplitudis deni aRiZvreba.

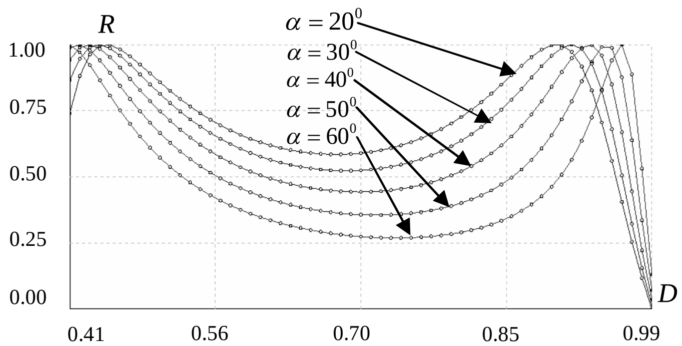


nax. 2.3.15 denis ganawileba elementSi

naxazze 2.3.16 moyvanilia arekvlis da gasvlis koeficientebis damokidebuleba D_2 periodze, D_1 periodis sxvadasxva mniSvnelobebisaTvis, rodesac R_0=0.2λ, $\alpha = 30^0$, N=20. interess warmoadgenda aseve arekvlis koeficientis damokidebuleba mesris periodze Ria seqtoris kuTxis sxvadasxva mniSvnelobebisaTvis. miRebuli damokidebuleba moyvanilia naxazze 2.3.17. am suraTis mixedviT Ria seqtoris kuTxis gaSla iwvevs rezonansis Seviwrovebas, rac Seesabameba sixSireze damokidebul zedapirs.



nax. 2.3.16 arekvlis da gasvlis koeficientis damokidebuleba mesris periodze. $R_0/\lambda = 0.2$, N=20



nax. 2.3.17 arekvlis koeficientis damokidebuleba mesris periodze, sxvadasxva gaSlis kuTxis SemTxvevaSi

daskvna

am TavSi ganixileboda brtyeli talRis difraqciis amocana organzomilebian usasrulo periodul meserze. moyvanilia am amocanis Teoriuli analizi da aseve ricxviTi eqsperimentebis Sedegebi.

Teoriul nawilSi iqna gamoyenebuli puasonis cnobili gardaqmna, romlis saSualebiT gabneuli veli warmodgenil iqna sivrculi, milevadi da aramilevadi speqtraluri komponentebis jamis saxiT. ricxviTi eqsperimentebis Catarebamde Semowmda algoriTmis cdomileba da dadginda damxmare parametrebis optimaluri mniSvnelobebi, mcire cdomilebis misaRebad. ganxilulia mesris elementis ori gansxvavebuli forma. gamokvleul iqna mesris gasvliTi da arekvliTi Tvisebebi. am TavSi miRebuli Sedegebi gamoqveynebul iqna statiis saxiT JurnalSi "Journal of Applied Electromagnetism" [46].

Tavi III

brtyeli eleqtromagnituri talRis difraqcia sistemaze usasrulo orperioduli meseri - brtyeli dieleqtrikli fena

zogadi mimoxilva

am TavSi amoxsnilia brtyeli eleqtromagnituri talRis difraqciis amocana sistemaze brtyeli usasrulo orperioduli meseri – brtyeli dieleqtrikuli fena [45, 46]. ganixileba ori gansxvavebuli SemTxveva, rodesac meseri imyofeba dieleqtrikis SigniT da aseve mis maxloblad. mesris elementi kvlav warmoadgens rezonansuli Tvisebebis mqone, brtyel wvril gamtars. zogadad mas SeiZleba gaaCndes rTuli forma. moyvanilia sami meTodi dasmuli amocanis amosaxsnelad.

difraqciis amocana dieleqtikSi moTavsebul periodul meserze dResdReobiT metad aqtualur problemas warmoadgens radgan aseTi saxis struqturebi, erTianobaSi garkveul sixSireebze, saintereso rezonansul Tvisebebs amJRavneben. es aixsneba imiT, rom rezonansuli elementebis gaerTianeba sistemaSi aZlierebs maT Soris urTierTqmedebas, rac iwvevs denis amplitudis mkveTr gazrdas yovel elementSi. aRniSnuli urTierTqmedeba damokidebulia agreTve manZilze elementebs Soris da es manZili aseve SeiZleba iyos rezonansuli. amasTanave, aRniSnuli rezonansuli efeqtebi damokidebulia agreTve mesris elementis geometriul formaze. rodesac sistema moTavsebulia naxevrad gamWvirvale dieleqtrikuli zedapiris SigniT, cxadia, moiZebneba sruli sistemis iseTi parametrebi, rodesac ganxiluli efeqtebi mkveTrad izrdeba da swored aseTi SemTxvevebis Seswavla warmoadgens metad saintereso amocanas.

mesame Tavi Sedgeba sam paragrafisgan:

pirvel paragrafSi ganixileba SemTxveva, rodesac meseri dieleqtrikis SigniT imyofeba. am amocanis amosaxsnelad gamoyenebulia damxmare gamomsxivseblebis meTodi. amisaTvis napovn iqna perioduli grinis funqcia, romelic iyo gamoyenebuli rogorc damxmare gamomsxiveblis veli. es niSnavs am meTodis ganviTarebas da morgebas aseTi saxis amocanebze.

meore paragrafSi ganixileba SemTxveva, rodesac meseri imyofeba fenis maxlobad. damxmare gamomsxiveblebis meTodTan erTad moyvanilia aseve am amocanis mkacri amoxsnis ori meTodi.

mesame paragrafSi moyvanilia miRebuli ricxviTi eqsperimentebis Sedegebi.

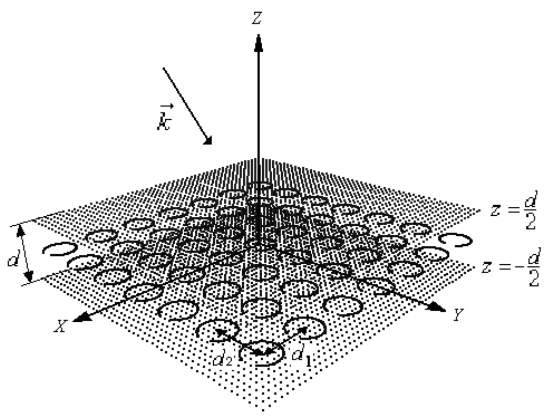
$3.1 brtyeli talRis difraqcia dieleqtrikul fenaSi moTavsebul usasrulo orperiodul meserze

amocanis dasma. ganvixiloT sasruli d sisqis mqone usasrulo dieleqtrikuli fena Tavisufal garemoSi, romelic XOY sibrtyis paraleluradaa orientirebuli. fena SemosazRvrulia $z=d/2$, $z=-d/2$ zedapirebiT da daxasiaTebulia ε dieleqtrikuli da μ magnituri SeRwevadobebiT. am fenis SigniT, $z=0$ sibrtyeSi imyofeba organzomilebiani usasrulo, d_1 da d_2 periodebis mqone meseri, romlis elementia, brtyeli mcire radiusis da rezonansuli Tvisebebis mqone gamtari (nax. 3.1.1).

aRniSnul struqturas $z>d/2$ naxevarsivrcidan ecema cnobili droSi harmoniuli brtyeli eleqtromagnituri talRa, romlis mimarTuleba $\vec{k}$ talRuri veqtoriT ganisazRvreba:

$$\vec{E}_{inc}(\vec{r})=\vec{E}_0 e^{i\vec{k}\vec{r}}, \quad \vec{H}_{inc}(\vec{r})=\vec{H}_0 e^{i\vec{k}\vec{r}}, \quad H_0=\sqrt{\varepsilon_0/\mu_0}E_0. \qquad (3.1.1)$$

aq $\vec{r}\{x,y,z\}$ dakvirvebis wertilis radiusveqtoria. drois maxasiaTebelia $e^{-i\omega t}$. saZiebelia gasuli veli, arekvlili veli da aseve veli struqturis SigniT.



nax. 3.1.1 meseri dieleqtrikSi

Cveni amocanaa, vipovoT difragirebuli veli struqturis zeviT, mis qveviT da aseve mis SigniT.

Ddamxmare gamomsxivebllebis meTodis gamoyeneba: ganxiluli amocanis Teoriulad gadawyvetisaTvis, moxerxebulia gamoyenebul iqnas damxmare gamomsxivebllebis meTodi. rogorc cnobilia, am meTodis gamoyenebis dros erTerT principul sakiTxs warmoadgens damxmare gamomsxivebllebis SerCeva, romlis maTematikuri gamosaxuleba unda akmayofilebdes saTanado amocanis diferencialur gantolebas da unda warmoadgendes am gantolebis grinis funqcias.

usasrulo perioduli mesris SemTxvevaSi, xdeba saWiro perioduli struqturis grinis funqciis gansazRvra. wina TavSi Cvens mier miRebul iqna usasrulo perioduli mesris mier gamosxivebuli velis gamosaxuleba. es gamosaxuleba iqna miRebuli mesris elementis dayofis gziT didi raodenobis mcire segmentebad. gasagebia, rom yovel konkretul segments mesris romelime elementSi, Seesabameba aseTive segmentebi mis mezobel elementebSi da aseTi msgavsi elementebis erToblioba qmnian imave periodebis mqone usasrulo meseრs. zustad aseTi segmentebisgan Semdgari mesris mier gamosxivebul vels unda vuwodoT perioduli grinis funqcia. maSasadame, damxmare gamomsxiveblebis meTodis gamoyenebis dros, unda davuSvaT, rom wertilovani damxmare wyaro asxivebs vels, romelic gamoisaxeba Semdegnairad:

$$\vec{G}_E\left(\vec{r},\vec{r}_\alpha\right)=\left(1/2\omega\varepsilon_0\varepsilon d_1 d_2\right)\sum_{m=-\infty}^{+\infty}\sum_{n=-\infty}^{+\infty}e^{i\vec{k}_{\alpha,mn}\left(\vec{r}-\vec{r}_\alpha\right)}\left(\mu\varepsilon k^2-k_{n,x}^2-k_{m,y}^2\right)^{-1/2}\left(\vec{k}_{\alpha,mn}\times\left(\vec{k}_{\alpha,mn}\times\vec{p}_\alpha\right)\right), \tag{3.1.2}$$

$$\vec{G}_H\left(\vec{r},\vec{r}_\alpha\right)=\left(1/2d_1 d_2\right)\sum_{n=-\infty}^{+\infty}\sum_{m=-\infty}^{+\infty}e^{i\vec{k}_{\alpha,mn}\left(\vec{r}-\vec{r}_\alpha\right)}\left(\mu\varepsilon k^2-k_{n,x}^2-k_{m,y}^2\right)^{-1/2}\left(\vec{p}_\alpha\times\vec{k}_{\alpha,mn}\right), \tag{3.1.3}$$

$$\vec{k}_{\alpha,mn}=\vec{k}_{\alpha,mn}\left\{k_{n,x},k_{m,y},\operatorname{sgn}\left(z-z_\alpha\right)\sqrt{\mu\varepsilon k^2-k_{n,x}^2-k_{m,y}^2}\right\},\ \ k_{n,x}=k_x+2\pi n/d_1,\ \ k_{m,y}=k_y+2\pi m/d_2 .$$

aq $\vec{r}_\alpha\left\{x_\alpha,y_\alpha,z_\alpha\right\}$ am damxmare gamomsxiveblis radiusveqtoria, $\vec{p}_\alpha$ erTeulovani veqtoria, romelic gansazRvravs am wyaros orientacias. aqve unda aRiniSnos, rom ganxilul grinis funqcias gaaCnia singularoba (ormagi mwkrivi ganSladia) sibrtyeSi $z=z_\alpha$, radgan fizikurad, es aris is sibrtye, saidanac vrceldebian brtyeli talRebi $z>z_\alpha$ da $z<z_\alpha$ naxevarsivrceebSi.

brtyeli dacemuli talRis da mesris usasrulobis gamo, ucnobi gabneuli veli yovel areSi aseve perioduli da yoveli periodis farglebSi erTi da igive amplitudis realuri nawili gaaCnia. amitom Cvens mier ganxilul iqneba mxolod sivrcis is nawili, romlis wertilebisaTvisac srulდeba Semdegi piroba: $\left(x,y\right)\in\left(d_1\times d_2\right)$.

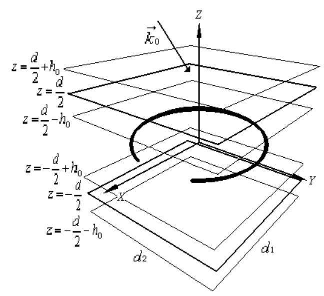
amocanis amoxsnis dros Cven dagvWirdeba aseve im velis gamosaxuleba, romelsac asxivebs meseri dieleqtrikis gareSe. es gamosaxuleba ukve iqna Cvens mier gamoyvanili da mas axla pirdapir moviyvanT:

$$\vec{E}\left(\vec{r}\right)=\sum_{j=1}^{N}I_j\,\vec{G}_E\left(\vec{r},\vec{r}_j\right)_{z_j=0,\vec{p}_j=d\vec{l}_j},\ \ \vec{H}\left(\vec{r}\right)=\sum_{j=1}^{N}I_j\,\vec{G}_H\left(\vec{r},\vec{r}_j\right)_{z_j=0,\vec{p}_j=d\vec{l}_j}. \tag{3.1.4}$$

axla gadavideT uSualod damxmare gamomsxiveblebis meTodis gamoyenebaze. avagoT dieleqtrikis SigniT da gareT, zeda da qveda zedapiridan mcire h_0 manZiliT daSorebuli oTxi damxmare zedapiri:

$$z=d/2-h_0,\ \ z=-d/2+h_0,\ \ z=d/2+h_0,\ \ z=-d/2-h_0 .$$

rogorc Cans, pirveli da meore damxmare zedapirebi struqturis SigniT imyofebian, xolo danarCeni ori – mis gareT.



nax. 3.1.2 ganxiluli are $\left(x,y\right)\in\left(d_1\times d_2\right)$

yvela damxmare zedapirze vaTavsebT zemoT aRniSnul damxmare gamomsxiveblebs. damxmare gamomsxiveblebiT Sida zedapirebze aRiwereba veli struqturis gareT, xolo gare damxmare zedapirebze mdebare gamomsxiveblebiT - veli struqturis SigniT. gasagebia, rom aq, gare damxmare wyaroebis velis garda arsebobs aseve mesris mier gabneuli veli. gamosakvlevi perioduli struqtura samganzomilebiania da amitom yoveli damxmare gamomsxivebeli unda Sedgebodes or urTierTmarTobul elementarul wyarosgan. es imas niSnavs, rom maTi orientacia unda ganisazRvrebodes Sesabamisad $\vec{x}$ da $\vec{y}$ bazisuri veqtorebiT. amasTanave, orive elementarul wyaros damxmare gamomsxivebelSi unda gaaCndes sakuTari ucnobi amplitudebi.

naTqvamis gaTvaliswinebiT davweroT zogadad difraqciis Sedegad warmoqmnili velebi.

are (I) – struqturis zeviT. aq gagvaCnia veli, romelic aRiwereba Sida $z=\dfrac{d}{2}-h_0$ damxmare zedapiriT da aseve dacemuli veli:

$$\vec{E}_{inc}(\vec{r})+\vec{E}_e(\vec{r})=\vec{E}_0 e^{i\vec{k}\vec{r}}+\sum_{\alpha=1}^{Q}\sum_{\alpha'=1}^{P}\left(A_{\alpha\alpha'}\underset{\mu=\varepsilon=1}{\vec{G}_E^x}(\vec{r},\vec{r}_{\alpha\alpha'})+B_{\alpha\alpha'}\underset{\mu=\varepsilon=1}{\vec{G}_E^y}(\vec{r},\vec{r}_{\alpha\alpha'})\right), \tag{3.1.5}$$

$$\vec{H}_{inc}(\vec{r})+\vec{H}_e(\vec{r})=\vec{H}_0 e^{i\vec{k}\vec{r}}+\sum_{\alpha=1}^{Q}\sum_{\alpha'=1}^{P}\left(A_{\alpha\alpha'}\underset{\mu=\varepsilon=1}{\vec{G}_H^x}(\vec{r},\vec{r}_{\alpha\alpha'})+B_{\alpha\alpha'}\underset{\mu=\varepsilon=1}{\vec{G}_H^y}(\vec{r},\vec{r}_{\alpha\alpha'})\right), \tag{3.1.6}$$

$$z_{\alpha\alpha'}=d/2-h_0,\quad z\in[d/2,+\infty),\quad \operatorname{sgn}(z-z_{\alpha\alpha'})=1.$$

aq $Q\times P$ damxmare gamomsxiveblebis raodenobaa erT damxmare zedapirze, $\alpha\alpha'$ indeqsebs warmoadgenen, romelnic gviCveneben am wyaros nomers, $A_{\alpha\alpha'}$ da $B_{\alpha\alpha'}$ misi ucnobi amplitudebia, x da y indeqsi gviCvenebs romel ortis gaswvrivaa orientirebuli misi elementaruli wyaro.

are (II) – struqturis SigniT. aq gagvaCnia $z=d/2+h_0$ da $z=-d/2-h_0$ damxmare zedapirebis mier Seqmnili veli da aseve mesris mier gabneuli veli:

$$\vec{E}(\vec{r})+\vec{E}_f(\vec{r})+\vec{E}_g(\vec{r})=\sum_{j=1}^{N}I_j\underset{z_j=0,\vec{p}_j=d\vec{l}_j}{\vec{G}_E}(\vec{r},\vec{r}_j)+$$
$$+\sum_{\beta=1}^{Q}\sum_{\beta'=1}^{P}\left(C_{\beta\beta'}\vec{G}_E^x(\vec{r},\vec{r}_{\beta\beta'})+D_{\beta\beta'}\vec{G}_E^y(\vec{r},\vec{r}_{\beta\beta'})\right)+\sum_{\gamma=1}^{Q}\sum_{\gamma'=1}^{P}\left(F_{\gamma\gamma'}\vec{G}_E^x(\vec{r},\vec{r}_{\gamma\gamma'})+L_{\gamma\gamma'}\vec{G}_E^y(\vec{r},\vec{r}_{\gamma\gamma'})\right), \tag{3.1.7}$$

$$\vec{H}(\vec{r})+\vec{H}_f(\vec{r})+\vec{H}_g(\vec{r})=\sum_{j=1}^{N}I_j\underset{z_j=0,\vec{p}_j=d\vec{l}_j}{\vec{G}_H}(\vec{r},\vec{r}_j)+$$
$$+\sum_{\beta=1}^{Q}\sum_{\beta'=1}^{P}\left(C_{\beta\beta'}\vec{G}_H^x(\vec{r},\vec{r}_{\beta\beta'})+D_{\beta\beta'}\vec{G}_H^y(\vec{r},\vec{r}_{\beta\beta'})\right)+\sum_{\gamma=1}^{Q}\sum_{\gamma'=1}^{P}\left(F_{\gamma\gamma'}\vec{G}_H^x(\vec{r},\vec{r}_{\gamma\gamma'})+L_{\gamma\gamma'}\vec{G}_H^y(\vec{r},\vec{r}_{\gamma\gamma'})\right), \tag{3.1.8}$$

$$z_{\beta\beta'}=d/2+h_0,\quad z_{\gamma\gamma'}=-d/2-h_0,\quad z\in[-d/2,d/2],\quad \operatorname{sgn}(z-z_{\beta\beta'})=-1,\quad \operatorname{sgn}(z-z_{\gamma\gamma'})=1.$$

are (III) – struqturis qveviT. aq gagvaCnia mxolod gasuli veli, romelic $z=-d/2+h_0$ zedapiriT aRiwereba:

$$\vec{E}_s(\vec{r})=\sum_{\delta=1}^{Q}\sum_{\delta'=1}^{P}\left(K_{\delta\delta'}\underset{\mu=\varepsilon=1}{\vec{G}_E^x}(\vec{r},\vec{r}_{\delta\delta'})+R_{\delta\delta'}\underset{\mu=\varepsilon=1}{\vec{G}_E^y}(\vec{r},\vec{r}_{\delta\delta'})\right), \tag{3.1.9}$$

$$\vec{H}_s(\vec{r}) = \sum_{\delta=1}^{Q}\sum_{\delta'=1}^{P}\left(K_{\delta\delta'}\, \underset{\mu=\varepsilon=1}{\vec{G}_H^x}(\vec{r},\vec{r}_{\delta\delta'}) + R_{\delta\delta'}\, \underset{\mu=\varepsilon=1}{\vec{G}_H^y}(\vec{r},\vec{r}_{\delta\delta'})\right), \quad (3.1.10)$$

$$z_{\delta\delta'} = -d/2 + h_0,\ \ z\in(-\infty,-d/2],\ \ \mathrm{sgn}(z - z_{\delta\delta'}) = -1.$$

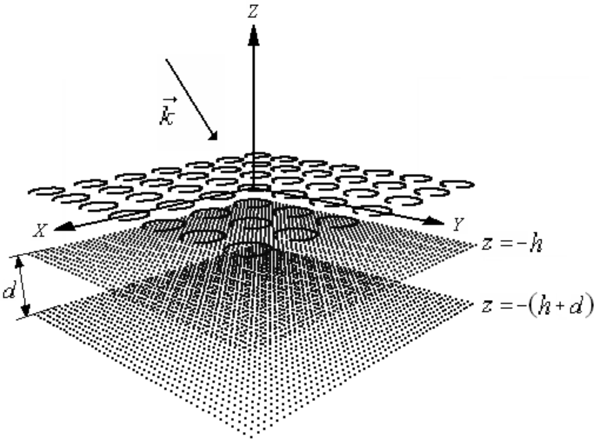
ucnobi amplitudebis gansazRvra. Cveni amocana dayvanilia imaze rom vipovoT denis ucnobi I_j amplitudebi ($j=1,2,...,N$) da damxmare gamomsxiveblebis amplitudebi $A_{\alpha\alpha'}$, $B_{\alpha\alpha'}$, $C_{\beta\beta'}$, $D_{\beta\beta'}$, $F_{\gamma\gamma'}$, $L_{\gamma\gamma'}$, $K_{\delta\delta'}$, $P_{\delta\delta'}$, sadac ($\alpha,\beta,\gamma,\delta=1,2,...,Q$, $\alpha',\beta',\gamma',\delta'=1,2,...,P$). sul gagvaCnia $8Q\times P+N$ ucnobi da isini sasazRvro pirobebidan unda vipovoT. dieleqtrikis zedapirze moviTxovT sasazRvro pirobis Sesrulebas $Q\times P$- cal gansxvavebul wertilSi. aseve viTxovT sasazRvro pirobis Sesrulebas mesris elementis N segmentze:

$$\begin{cases}
\left(\vec{E}_{inc}(\vec{r}_{\varphi\varphi'}) + \vec{E}_e(\vec{r}_{\varphi\varphi'})\right)_{z_{\varphi\varphi'}=d/2}\cdot\vec{x} = \left(\vec{E}(\vec{r}_{\varphi\varphi'}) + \vec{E}_f(\vec{r}_{\varphi\varphi'}) + \vec{E}_g(\vec{r}_{\varphi\varphi'})\right)_{z_{\varphi\varphi'}=d/2}\cdot\vec{x} \\
\left(\vec{E}_{inc}(\vec{r}_{\varphi\varphi'}) + \vec{E}_e(\vec{r}_{\varphi\varphi'})\right)_{z_{\varphi\varphi'}=d/2}\cdot\vec{y} = \left(\vec{E}(\vec{r}_{\varphi\varphi'}) + \vec{E}_f(\vec{r}_{\varphi\varphi'}) + \vec{E}_g(\vec{r}_{\varphi\varphi'})\right)_{z_{\varphi\varphi'}=d/2}\cdot\vec{y} \\
\left(\vec{E}(\vec{r}_{\varphi\varphi'}) + \vec{E}_f(\vec{r}_{\varphi\varphi'}) + \vec{E}_g(\vec{r}_{\varphi\varphi'})\right)_{z_{\varphi\varphi'}=-d/2}\cdot\vec{x} = \vec{E}_s(\vec{r}_{\varphi\varphi'})_{z_{\varphi\varphi'}=-d/2}\cdot\vec{x} \\
\left(\vec{E}(\vec{r}_{\varphi\varphi'}) + \vec{E}_f(\vec{r}_{\varphi\varphi'}) + \vec{E}_g(\vec{r}_{\varphi\varphi'})\right)_{z_{\varphi\varphi'}=-d/2}\cdot\vec{y} = \vec{E}_s(\vec{r}_{\varphi\varphi'})_{z_{\varphi\varphi'}=-d/2}\cdot\vec{y} \\
\left(\vec{H}_{inc}(\vec{r}_{\varphi\varphi'}) + \vec{H}_e(\vec{r}_{\varphi\varphi'})\right)_{z_{\varphi\varphi'}=d/2}\cdot\vec{x} = \left(\vec{H}(\vec{r}_{\varphi\varphi'}) + \vec{H}_f(\vec{r}_{\varphi\varphi'}) + \vec{H}_g(\vec{r}_{\varphi\varphi'})\right)_{z_{\varphi\varphi'}=d/2}\cdot\vec{x} \\
\left(\vec{H}_{inc}(\vec{r}_{\varphi\varphi'}) + \vec{H}_e(\vec{r}_{\varphi\varphi'})\right)_{z_{\varphi\varphi'}=d/2}\cdot\vec{y} = \left(\vec{H}(\vec{r}_{\varphi\varphi'}) + \vec{H}_f(\vec{r}_{\varphi\varphi'}) + \vec{H}_g(\vec{r}_{\varphi\varphi'})\right)_{z_{\varphi\varphi'}=d/2}\cdot\vec{y} \\
\left(\vec{H}(\vec{r}_{\varphi\varphi'}) + \vec{H}_f(\vec{r}_{\varphi\varphi'}) + \vec{H}_g(\vec{r}_{\varphi\varphi'})\right)_{z_{\varphi\varphi'}=-d/2}\cdot\vec{x} = \vec{H}_s(\vec{r}_{\varphi\varphi'})_{z_{\varphi\varphi'}=-d/2}\cdot\vec{x} \\
\left(\vec{H}(\vec{r}_{\varphi\varphi'}) + \vec{H}_f(\vec{r}_{\varphi\varphi'}) + \vec{H}_g(\vec{r}_{\varphi\varphi'})\right)_{z_{\varphi\varphi'}=-d/2}\cdot\vec{y} = \vec{H}_s(\vec{r}_{\varphi\varphi'})_{z_{\varphi\varphi'}=-d/2}\cdot\vec{y} \\
\left(\vec{E}(\vec{r}_\sigma) + \vec{E}_f(\vec{r}_\sigma) + \vec{E}_g(\vec{r}_\sigma)\right)\cdot d\vec{l}_\sigma = 0,
\end{cases}$$

sadac $\varphi=1,2,...,Q$, $\varphi'=1,2,...,P$ $\sigma=1,2,...,N$. am sistemis amoxsna kompiuteruli modelirebiT xdeba. amis Semdeg SegviZlia vipovoT difraqciis Sedegad miRebuli veli sivrcis nebismier wertilSi (3.1.5) – (3.1.10) formulebis saSualebiT.

§3.2 brtyeli talRis difraqcia dieleqtrikuli fenis maxloblad moTavsebul usasrulo orperiodul meserze

amocanis dasma. ganvixiloT d sisqis da ε, μ SeRwevadobebis mqone brtyeli dieleqtrikuli fena. perioduli meseri, romlis periodebia d_1 da d_2 imyofeba fenisgan h simaRleze da misi yvela elementi erT sibrtyeSi imyofeba (nax. 3.2.1).



nax. 3.2.1 sistemis geometria

meserS ecema cnobili, droSi harmoniuli brtyeli eleqtromagnituri talRa

$$\vec{E}_{inc}(\vec{r})=\vec{E}_0 e^{i\vec{k}\vec{r}},\ \vec{H}_{inc}(\vec{r})=\vec{H}_0 e^{i\vec{k}\vec{r}}, \quad (3.2.1)$$

sadac $\vec{r}=\vec{r}\{x,y,z\}$ dakvirvebis wertilis radiusveqtoria, $\vec{k}=\vec{k}\{k_x,k_y,k_z\}$ talRuri veqtoria, $k=\omega\sqrt{\varepsilon_0\mu_0}$. droze damokidebuleba gamoisaxeba rogorc $e^{-i\omega t}$. Cveni amocanaa vipovoT difragirebuli velebi Semdeg areebSi: (I)-mesris zeviT, (II)-meserSa da dieleqtrikuli fenis Soris, (III)-dieleqtrikuli fenis SigniT, (IV)-dieleqtrikuli fenis qveviT.

radgan, meseri usasrulo periodulia da dacemuli talRa aris brtyeli, aqedan gamomdinare difragirebul velebs yvela areSi sivrculi perioduloba unda gaaCndeT da amitom Cvens mier ganxilul iqneba mxolod erTi aseTi periodi, anu sivrcis is nawili, romlis wertilebis koordinatebisaTvis srulდeba pirobebi

$$x\in[-d_1/2,d_1/2],\ y\in[-d_2/2,d_2/2],\ z\in(-\infty,+\infty).$$

gasagebia, rom sivrcis es nawili mxolod mesris centralur (nulovan) elements Seicavs.

SemoviRoT sakoordinato sistema ise, rom meseri imyofebodes XOY sibrtyeSi, xolo dieleqtrikuli fenis zedapirebs warmoadgendnen $z=-h$ da $z=-(h+d)$ sibrtyeebi. mesris nulovani elementis eleqtrulad mcire radiusi avRniSnoT rogorc dr_0, xolo am elementis centraluri wiris gantoleba parametruli saxiT SemoviRoT:

$$x_0=x_0(t),\ y_0=y_0(t),\ z_0=z_0(t)\equiv 0,\ t\in[t_1,t_2].$$

pirvel areSi gagvaCnia dacemuli veli da aseve mesrisgan zeviT mimavali velebi:

$$\text{(I)}:\ \vec{E}_{inc}(\vec{r})+\vec{E}_1(\vec{r}),\ \vec{H}_{inc}(\vec{r})+\vec{H}_1(\vec{r}),\ z>dr_0. \quad (3.2.2)$$

analogiurad, meore areSi gagvaCnia mesrisgan qveviT mimavali velebi da aseve $z=-h$ zedapiridan arekvlili velebi:

$$\text{(II)}:\ \vec{E}_2(\vec{r})+\vec{E}_e(\vec{r}),\ \vec{H}_2(\vec{r})+\vec{H}_e(\vec{r}),\ -h<z<-dr_0. \quad (3.2.3)$$

mesame areSi gagvaCnia $z=-h$ zedapiridan gardatexili velebi da aseve $z=-(h+d)$ zedapiridan zeviT arekvlili velebi:

$$\text{(III)}:\ \vec{E}_f(\vec{r})+\vec{E}_g(\vec{r}),\ \vec{H}_f(\vec{r})+\vec{H}_g(\vec{r}),\ -(h+d)<z<-h. \qquad (3.2.4)$$

meoTxe areSi gagvaCnia mxolod $z=-(h+d)$ zedapiridan qveviT mimavali (gasuli) velebi:

$$\text{(IV)}:\ \vec{E}_s(\vec{r}),\ \vec{H}_s(\vec{r}),\ z<-(h+d). \qquad (3.2.5)$$

Cven ar ganvixilavT Txel fenas $-dr_0 \le z \le dr_0$ romelSic mesridan wamosul velebs singularoba gaaCniaT.

aRniSnuli velebi unda akmayofilebdnen sasazRvro pirobebs dieleqtrikuli fenis orive zedapirze da aseve mesris elementebis gaswvriv. am pirobebis raodenoba udris cxras. erTi piroba iwereba mesris elementis zedapirze, ris gamoc mas lokaluri xasiaTi gaaCnia da igi mdgomareobs jamuri eleqtruli velis tangencialuri mdgenelis nulTan tolobaSi:

$$\left(\vec{E}_{inc}(\vec{r}_\sigma)+\vec{E}_1(\vec{r}_\sigma)+\vec{E}_e(\vec{r}_\sigma)\right)\cdot\vec{\tau}=0. \qquad (3.2.6)$$

aq $\vec{r}_\sigma$ aris elementis zedapirze aRebuli wertilis radiusveqtori, xolo $\vec{\tau}$ warmoadgens am wertilSi gavlebul tangencials. radgan mesris elementi wvrilia, Cven ar ganvixilavT im tangencials romelsac radialuri mimarTuleba gaaCnia.

danarCeni rva piroba unda exebodes dieleqtrikuli fenis zedapirebs, romelic araa lokaluri da zedapiris yvela wertilSi unda sruldebodes. gagvaCnia ori zedapiri da Sesabamisad yovel maTganze oTxi sasazRvro pirobaa dasaweri. am oTxi pirobidan ori unda daiweros eleqtruli velisTvis, xolo danarCeni ori – magniturisTvis (radgan dieleqtrikis zedapirs ori wrfivad damoukidebeli tangenciali gaaCnia). rogorc viciT, dieleqtrikis zedapirze moiTxoveba daZabulobis veqtoris tangencialuri mdgenelis uwyvetobis piroba. maSasadame

$$\begin{cases}
\left(\vec{E}_2(\vec{r})+\vec{E}_e(\vec{r})\right)\Big|_{z=-h}\cdot\vec{x}=\left(\vec{E}_f(\vec{r})+\vec{E}_g(\vec{r})\right)\Big|_{z=-h}\cdot\vec{x}\\
\left(\vec{H}_2(\vec{r})+\vec{H}_e(\vec{r})\right)\Big|_{z=-h}\cdot\vec{x}=\left(\vec{H}_f(\vec{r})+\vec{H}_g(\vec{r})\right)\Big|_{z=-h}\cdot\vec{x}\\
\left(\vec{E}_2(\vec{r})+\vec{E}_e(\vec{r})\right)\Big|_{z=-h}\cdot\vec{y}=\left(\vec{E}_f(\vec{r})+\vec{E}_g(\vec{r})\right)\Big|_{z=-h}\cdot\vec{y}\\
\left(\vec{H}_2(\vec{r})+\vec{H}_e(\vec{r})\right)\Big|_{z=-h}\cdot\vec{y}=\left(\vec{H}_f(\vec{r})+\vec{H}_g(\vec{r})\right)\Big|_{z=-h}\cdot\vec{y}\\
\left(\vec{E}_f(\vec{r})+\vec{E}_g(\vec{r})\right)\Big|_{z=-(h+d)}\cdot\vec{x}=\vec{E}_s(\vec{r})\Big|_{z=-(h+d)}\cdot\vec{x}\\
\left(\vec{H}_f(\vec{r})+\vec{H}_g(\vec{r})\right)\Big|_{z=-(h+d)}\cdot\vec{x}=\vec{H}_s(\vec{r})\Big|_{z=-(h+d)}\cdot\vec{x}\\
\left(\vec{E}_f(\vec{r})+\vec{E}_g(\vec{r})\right)\Big|_{z=-(h+d)}\cdot\vec{y}=\vec{E}_s(\vec{r})\Big|_{z=-(h+d)}\cdot\vec{y}\\
\left(\vec{H}_f(\vec{r})+\vec{H}_g(\vec{r})\right)\Big|_{z=-(h+d)}\cdot\vec{y}=\vec{H}_s(\vec{r})\Big|_{z=-(h+d)}\cdot\vec{y}
\end{cases}. \qquad (3.2.7)$$

amocanis amoxsna pirveli meTodiT. dasmuli amocanis amoxsnis dros Cven dagvWirdeba im velis gamosaxuleba, romelsac meseri asxivebs dieleqtrikulo fenis gareSe. es gamosaxuleba warmoadgens brtyeli milevadi da aramilevadi talRebis jams. igi iqna napovni naSromis wina TavSi da amitom mas moviyvanT gamoyvanis gareSe. aq mxolod gavixsenebT, rom igi miRebulia mesris elementis warmodgeniT rogorc didi N raodenobis mcire dl sigrZis mqone segmentebis erToblioba ($\left|d\vec{l}_j\right| = dl$, $j=1,2,...,N$), $\vec{r}_j$ warmoadgens j segmentis radiusveqtors, A_j ucnobi koeficientebia da yoveli maTgani, fizikurad warmoadgens Sesabamis segmentSi aRZrul dens.

Cven cal-calke ganvixilavT zeviT da qveviT mimaval velebs. amitom:

$$\vec{E}_1(\vec{r}) = \left(1/2\omega\varepsilon_0 d_1 d_2\right)\sum_{j=1}^{N} A_j \sum_{m=-\infty}^{+\infty}\sum_{n=-\infty}^{+\infty} k_{mn,z}^{-1} e^{i\vec{k}_{mn}^{1}\cdot(\vec{r}-\vec{r}_j)} \vec{P}_{j,mn}^{1},$$

$$\vec{H}_1(\vec{r}) = \left(1/2 d_1 d_2\right)\sum_{j=1}^{N} A_j \sum_{m=-\infty}^{+\infty}\sum_{n=-\infty}^{+\infty} k_{mn,z}^{-1} e^{i\vec{k}_{mn}^{1}\cdot(\vec{r}-\vec{r}_j)} \left(d\vec{l}_j \times \vec{k}_{mn}^{1}\right),$$

$$\vec{P}_{j,mn}^{1} = \vec{k}_{mn}^{1} \times \left(\vec{k}_{mn}^{1} \times d\vec{l}_j\right), \quad \vec{k}_{mn}^{1} = \vec{k}_{mn}^{1}\left\{k_{n,x}, k_{m,y}, k_{mn,z}\right\}$$

da aseve

$$\vec{E}_2(\vec{r}) = \left(1/2\omega\varepsilon_0 d_1 d_2\right)\sum_{j=1}^{N} A_j \sum_{m=-\infty}^{+\infty}\sum_{n=-\infty}^{+\infty} k_{mn,z}^{-1} e^{i\vec{k}_{mn}^{2}\cdot(\vec{r}-\vec{r}_j)} \vec{P}_{j,mn}^{2},$$

$$\vec{H}_2(\vec{r}) = \left(1/2 d_1 d_2\right)\sum_{j=1}^{N} A_j \sum_{m=-\infty}^{+\infty}\sum_{n=-\infty}^{+\infty} k_{mn,z}^{-1} e^{i\vec{k}_{mn}^{2}\cdot(\vec{r}-\vec{r}_j)} \left(d\vec{l}_j \times \vec{k}_{mn}^{2}\right),$$

$$\vec{P}_{j,mn}^{2} = \vec{k}_{mn}^{2} \times \left(\vec{k}_{mn}^{2} \times d\vec{l}_j\right), \quad \vec{k}_{mn}^{2} = \vec{k}_{mn}^{2}\left\{k_{n,x}, k_{m,y}, -k_{mn,z}\right\}.$$

am gamosaxulebebSi

$$d\vec{l}_j = d\vec{l}_j\left\{dx_j, dy_j, 0\right\}, \left|\vec{k}_{mn}^{1}\right| = \left|\vec{k}_{mn}^{2}\right| = k,$$

$$k_{n,x} = k_x + 2\pi n/d_1, k_{m,y} = k_y + 2\pi m/d_2, \quad k_{mn,z} = \sqrt{k^2 - k_{n,x}^2 - k_{m,y}^2}.$$

gadavideT axla danarCeni velebis gansazRvraze yvela areSi. pirvel areSi gagvaCnia $\vec{E}_1(\vec{r})$ da dacemuli $\vec{E}_{inc}(\vec{r})$ velebi (3.2.2).

$$\vec{E}_1(\vec{r}) + \vec{E}_{inc}(\vec{r}) = \left(1/2\omega\varepsilon_0 d_1 d_2\right)\sum_{j=1}^{N} A_j \sum_{m=-\infty}^{+\infty}\sum_{n=-\infty}^{+\infty} k_{mn,z}^{-1} e^{i\vec{k}_{mn}^{1}\cdot(\vec{r}-\vec{r}_j)} \vec{P}_{j,mn}^{1} + \vec{E}_{inc}(\vec{r}). \tag{3.2.8}$$

unda aRiniSnos, rom dieleqtrikidan mesrisaken wamosuli harmonikis polarizacia ganisazRvreba $\vec{P}_{j,mn}^{2}$ veqtoriT, anu $d\vec{l}_j$ da $\vec{k}_{mn}^{2}$ veqtorebiT:

$$\vec{P}_{j,mn}^{2} = \vec{k}_{mn}^{2} \times \left(\vec{k}_{mn}^{2} \times d\vec{l}_j\right) = \vec{k}_{mn}^{2}\left(\vec{k}_{mn}^{2} \cdot d\vec{l}_j\right) - k^2 d\vec{l}_j.$$

dieleqtrikul fenasTan urTierTqmedebis Semdeg miRebuli velebis polarizaciis veqtorebi unda imyofebodnen imave sibrtyeSi. amis gamo, maTi orientacia kvlav unda ganisazRvrebodes $d\vec{l}_j$ da $\vec{k}_{mn}^{2}$ veqtorebiT.

aRniSnulis gaTvaliswinebiT, meore areSi velis (3.2.3) gamosaxuleba veZeboT rogorc

$$\vec{E}_2(\vec{r}) + \vec{E}_e(\vec{r}) = \left(1/2\omega\varepsilon_0 d_1 d_2\right)\sum_{j=1}^{N} A_j \sum_{m=-\infty}^{+\infty}\sum_{n=-\infty}^{+\infty} k_{mn,z}^{-1}\left(e^{i\vec{k}_{mn}^{2}\cdot(\vec{r}-\vec{r}_j)} \vec{P}_{j,mn}^{2} + e^{i\vec{k}_{mn}^{1}\cdot(\vec{r}-\vec{r}_j)} \vec{R}_{j,mn}\right), \tag{3.2.9}$$

sadac

$$\vec{R}_{j,mn} = B_{j,mn}\vec{k}_{mn}^{2} + C_{j,mn} d\vec{l}_j ,$$

$B_{j,mn}$ da $C_{j,mn}$ ucnobi koeficientebia.

mesame areSi gagvaCnia $z=-h$ zedapiridan gardatexili da $z=-(h+d)$ zedapiridan arekvlili velebi (3.2.4), romlebsac veZebT Semdegi saxiT:

$$\vec{E}_f(\vec{r}) + \vec{E}_g(\vec{r}) = \left(1/2\omega\varepsilon_0\varepsilon d_1 d_2\right)\sum_{j=1}^{N} A_j \sum_{m=-\infty}^{+\infty}\sum_{n=-\infty}^{+\infty} k'^{-1}_{mn,z}\left(e^{i\vec{k}^{f}_{mn}\cdot(\vec{r}-\vec{r}_j)}\vec{Q}_{j,mn} + e^{i\vec{k}^{g}_{mn}\cdot(\vec{r}-\vec{r}_j)}\vec{S}_{j,mn}\right), \quad (3.2.10)$$

sadac

$$\vec{k}^{f}_{mn} = \vec{k}^{f}_{mn}\left\{k_{n,x}, k_{m,y}, -k'_{mn,z}\right\},\ \ \vec{k}^{g}_{mn} = \vec{k}^{g}_{mn}\left\{k_{n,x}, k_{m,y}, k'_{mn,z}\right\},\ \ k'_{mn,z} = \sqrt{k^2\mu\varepsilon - k_{n,x}^2 - k_{m,y}^2},$$

$$\vec{Q}_{j,mn} = D_{j,mn}\vec{k}^{2}_{mn} + F_{j,mn} d\vec{l}_j ,\ \ \vec{S}_{j,mn} = G_{j,mn}\vec{k}^{2}_{mn} + L_{j,mn} d\vec{l}_j ,$$

$D_{j,mn}$, $F_{j,mn}$, $G_{j,mn}$, $L_{j,mn}$ ucnobi koeficientebia.

meoTxe areSi gagvaCnia mxolod sistemidan gamosuli velebi (3.2.5), romelsac veZebT kvlav rogorc:

$$\vec{E}_s(\vec{r}) = \left(1/2\omega\varepsilon_0 d_1 d_2\right)\sum_{j=1}^{N} A_j \sum_{m=-\infty}^{+\infty}\sum_{n=-\infty}^{+\infty} k^{-1}_{mn,z} e^{i\vec{k}^{2}_{mn}\cdot(\vec{r}-\vec{r}_j)}\vec{T}_{j,mn} , \quad (3.2.11)$$

sadac

$$\vec{T}_{j,mn} = \Gamma_{j,mn}\vec{k}^{2}_{mn} + Y_{j,mn} d\vec{l}_j ,$$

xolo $\Gamma_{j,mn}$, $Y_{j,mn}$ ucnob koeficientebs warmoadgenen.

magnitur vels yvela areSi vipoviT maqsvelis gantolebidan

$$\vec{H}(\vec{r}) = -\left(i/\omega\mu_0\mu\right) rot\vec{E}(\vec{r}).$$

miviRebT:

$$\vec{H}_1(\vec{r}) + \vec{H}_{inc}(\vec{r}) = \left(1/2d_1 d_2\right)\sum_{j=1}^{N} A_j \sum_{m=-\infty}^{+\infty}\sum_{n=-\infty}^{+\infty} k^{-1}_{mn,z} e^{i\vec{k}^{1}_{mn}\cdot(\vec{r}-\vec{r}_j)}\left(d\vec{l}_j \times \vec{k}^{1}_{mn}\right) + \vec{H}_{inc}(\vec{r}), \quad (3.2.12)$$

$$\vec{H}_2(\vec{r}) + \vec{H}_e(\vec{r}) = \left(1/2d_1 d_2\right)\cdot$$
$$\cdot\sum_{j=1}^{N} A_j \sum_{m=-\infty}^{+\infty}\sum_{n=-\infty}^{+\infty} k^{-1}_{mn,z}\left(e^{i\vec{k}^{2}_{mn}\cdot(\vec{r}-\vec{r}_j)}\left(d\vec{l}_j \times \vec{k}^{2}_{mn}\right) - \left(k^2\right)^{-1} e^{i\vec{k}^{1}_{mn}\cdot(\vec{r}-\vec{r}_j)}\left(\vec{R}_{j,mn} \times \vec{k}^{1}_{mn}\right)\right), \quad (3.2.13)$$

$$\vec{H}_f(\vec{r}) + \vec{H}_g(\vec{r}) = -\left(1/2d_1 d_2 k^2\mu\varepsilon\right)\cdot$$
$$\cdot\sum_{j=1}^{N} A_j \sum_{m=-\infty}^{+\infty}\sum_{n=-\infty}^{+\infty} k'^{-1}_{mn,z}\left(e^{i\vec{k}^{f}_{mn}\cdot(\vec{r}-\vec{r}_j)}\left(\vec{Q}_{j,mn} \times \vec{k}^{f}_{mn}\right) + e^{i\vec{k}^{g}_{mn}\cdot(\vec{r}-\vec{r}_j)}\left(\vec{S}_{j,mn} \times \vec{k}^{g}_{mn}\right)\right), \quad (3.2.14)$$

$$\vec{H}_s(\vec{r}) = -\left(1/2d_1 d_2 k^2\right)\sum_{j=1}^{N} A_j \sum_{m=-\infty}^{+\infty}\sum_{n=-\infty}^{+\infty} k^{-1}_{mn,z} e^{i\vec{k}^{2}_{mn}\cdot(\vec{r}-\vec{r}_j)}\left(\vec{T}_{j,mn} \times \vec{k}^{2}_{mn}\right). \quad (3.2.15)$$

maSasadame, gagvaCnia Semdegi ucnobi koeficientebi: A_j, $B_{j,mn}$, $C_{j,mn}$, $D_{j,mn}$, $F_{j,mn}$, $G_{j,mn}$, $L_{j,mn}$, $\Gamma_{j,mn}$, $Y_{j,mn}$ da maTi gansazRvra sasazRvro pirobebidan SeiZleba.

CavsvaT exla velis (3.2.8) – (3.2.15) gamosaxulebebi (3.2.7) sasazRvro pirobebSi dieleqtrikis zedapirebze. pirvel pirobidan miviRebT:

$$\sum_{j=1}^{N} A_j \sum_{m=-\infty}^{+\infty}\sum_{n=-\infty}^{+\infty} k_{mn,z} e^{i\left(k_{n,x}(x-x_j)+k_{m,y}(y-y_j)\right)}\left(e^{ik_{mn,z}h}\vec{P}^{2}_{j,mn} + e^{-ik_{mn,z}h}\vec{R}_{j,mn}\right)\cdot\vec{x} =$$

$$= \sum_{j=1}^{N} A_j \sum_{m=-\infty}^{+\infty}\sum_{n=-\infty}^{+\infty}\left(\varepsilon k'_{mn,z}\right)^{-1} e^{i\left(k_{n,x}(x-x_j)+k_{m,y}(y-y_j)\right)}\left(e^{ik'_{mn,z}h}\vec{Q}_{j,mn} + e^{-ik'_{mn,z}h}\vec{S}_{j,mn}\right)\cdot\vec{x} .$$

es toloba unda iyos samarTliani nebismieri (x,y) wyvilisaTvis. mas mniSvnelovnad gavamartivebT, Tu gaviTvaliswinebT, rom funqciebi $e^{i\left(k_{n,x}\left(x-x_j\right)+k_{m,y}\left(y-y_j\right)\right)}$ hqmnian orTogonalur sistemas $\Delta_{d_1\times d_2}$ marTkuTxedis farglebSi, sadac

$$\Delta_{d_1\times d_2}=\left[-d_1/2+x_j,d_1/2+x_j\right]\cup\left[-d_2/2+y_j,d_2/2+y_j\right].$$

amisaTvis tolobis orive mxare gavamravloT $e^{-i\left(k_{p,x}\left(x-x_j\right)+k_{q,y}\left(y-y_j\right)\right)}$- ze, (sadac p da q fiqsirebuli mTeli ricxvebia) da Semdeg gavaintegroT $\Delta_{d_1\times d_2}$ areSi:

$$\sum_{j=1}^{N}A_j\sum_{m=-\infty}^{+\infty}\sum_{n=-\infty}^{+\infty}\iint_{\Delta_{d_1\times d_2}}e^{i\left(\left(k_{n,x}-k_{p,x}\right)\left(x-x_j\right)+\left(k_{m,y}-k_{q,y}\right)\left(y-y_j\right)\right)}dxdy\times k_{mn,z}^{-1}\left(e^{ik_{mn,z}h}\vec{P}_{j,mn}^{2}+e^{-ik_{mn,z}h}\vec{R}_{j,mn}\right)\cdot\vec{x}=$$

$$=\sum_{j=1}^{N}A_j\sum_{m=-\infty}^{+\infty}\sum_{n=-\infty}^{+\infty}\iint_{\Delta_{d_1\times d_2}}e^{i\left(\left(k_{n,x}-k_{p,x}\right)\left(x-x_j\right)+\left(k_{m,y}-k_{q,y}\right)\left(y-y_j\right)\right)}dxdy\times\left(\varepsilon k'_{mn,z}\right)^{-1}\left(e^{ik'_{mn,z}h}\vec{Q}_{j,mn}+e^{-ik'_{mn,z}h}\vec{S}_{j,mn}\right)\cdot\vec{x}\,.$$

integrirebis Sedegad vRebulobT, rom

$$\iint_{\Delta_{d_1\times d_2}}e^{i\left(\left(k_{n,x}-k_{p,x}\right)\left(x-x_j\right)+\left(k_{m,y}-k_{q,y}\right)\left(y-y_j\right)\right)}dxdy=\frac{d_1d_2\sin\pi\left(n-p\right)\sin\pi\left(m-q\right)}{\pi^2\left(n-p\right)\left(m-q\right)}=\begin{cases}0,n\neq p,\left(m\neq q\right)\\ d_1d_2,n=p,m=q\end{cases}.$$

es imas niSnavs, rom tolobis orive mxareSi, ormagi usasrulo jamisgan dagvrCeba mxolod is wevri, romlisaTvis $n=p$ da $m=q$. miviRebT:

$$\sum_{j=1}^{N}A_j\left(k_{qp,z}^{-1}\left(e^{ik_{qp,z}h}\vec{P}_{j,qp}^{2}+e^{-ik_{qp,z}h}\vec{R}_{j,qp}\right)-\left(\varepsilon k'_{qp,z}\right)^{-1}\left(e^{ik'_{qp,z}h}\vec{Q}_{j,qp}+e^{-ik'_{qp,z}h}\vec{S}_{j,qp}\right)\right)\cdot\vec{x}=0\,.$$

analogiurad, danarCen sasazRvro pirobebidan, gveqneba:

$$\sum_{j=1}^{N}A_j\left(k_{qp,z}^{-1}\left(\left(k^2\right)^{-1}e^{-ik_{qp,z}h}\left(\vec{R}_{j,qp}\times\vec{k}_{qp}^{1}\right)-e^{ik_{qp,z}h}\left(d\vec{l}_j\times\vec{k}_{qp}^{2}\right)\right)-\right.$$

$$\left.-\left(k^2\mu\varepsilon k'_{qp,z}\right)^{-1}\left(e^{ik'_{qp,z}h}\left(\vec{Q}_{j,qp}\times\vec{k}_{qp}^{f}\right)+e^{-ik'_{qp,z}h}\left(\vec{S}_{j,qp}\times\vec{k}_{qp}^{g}\right)\right)\right)\cdot\vec{x}=0,$$

$$\sum_{j=1}^{N}A_j\left(k_{qp,z}^{-1}\left(e^{ik_{qp,z}h}\vec{P}_{j,qp}^{2}+e^{-ik_{qp,z}h}\vec{R}_{j,qp}\right)-\left(\varepsilon k'_{qp,z}\right)^{-1}\left(e^{ik'_{qp,z}h}\vec{Q}_{j,qp}+e^{-ik'_{qp,z}h}\vec{S}_{j,qp}\right)\right)\cdot\vec{y}=0\,,$$

$$\sum_{j=1}^{N}A_j\left(k_{qp,z}^{-1}\left(\left(k^2\right)^{-1}e^{-ik_{qp,z}h}\left(\vec{R}_{j,qp}\times\vec{k}_{qp}^{1}\right)-e^{ik_{qp,z}h}\left(d\vec{l}_j\times\vec{k}_{qp}^{2}\right)\right)-\right.$$

$$\left.-\left(k^2\mu\varepsilon k'_{qp,z}\right)^{-1}\left(e^{ik'_{qp,z}h}\left(\vec{Q}_{j,qp}\times\vec{k}_{qp}^{f}\right)+e^{-ik'_{qp,z}h}\left(\vec{S}_{j,qp}\times\vec{k}_{qp}^{g}\right)\right)\right)\cdot\vec{y}=0,$$

$$\sum_{j=1}^{N}A_j\left(\left(\varepsilon k'_{qp,z}\right)^{-1}\left(e^{ik'_{qp,z}(h+d)}\vec{Q}_{j,qp}+e^{-ik'_{qp,z}(h+d)}\vec{S}_{j,qp}\right)-k_{qp,z}^{-1}e^{ik_{qp,z}(h+d)}\vec{T}_{j,qp}\right)\cdot\vec{x}=0\,,$$

$$\sum_{j=1}^{N}A_j\left(\left(\mu\varepsilon k'_{qp,z}\right)^{-1}\left(e^{ik'_{qp,z}(h+d)}\left(\vec{Q}_{j,qp}\times\vec{k}_{qp}^{f}\right)+e^{-ik'_{qp,z}(h+d)}\left(\vec{S}_{j,qp}\times\vec{k}_{qp}^{g}\right)\right)-k_{qp,z}^{-1}e^{ik_{qp,z}(h+d)}\left(\vec{T}_{j,qp}\times\vec{k}_{qp}^{2}\right)\right)\cdot\vec{x}=0\,,$$

$$\sum_{j=1}^{N}A_j\left(\left(\varepsilon k'_{qp,z}\right)^{-1}\left(e^{ik'_{qp,z}(h+d)}\vec{Q}_{j,qp}+e^{-ik'_{qp,z}(h+d)}\vec{S}_{j,qp}\right)-k_{qp,z}^{-1}e^{ik_{qp,z}(h+d)}\vec{T}_{j,qp}\right)\cdot\vec{y}=0\,,$$

$$\sum_{j=1}^{N}A_j\left(\left(\mu\varepsilon k'_{qp,z}\right)^{-1}\left(e^{ik'_{qp,z}(h+d)}\left(\vec{Q}_{j,qp}\times\vec{k}_{qp}^{f}\right)+e^{-ik'_{qp,z}(h+d)}\left(\vec{S}_{j,qp}\times\vec{k}_{qp}^{g}\right)\right)-k_{qp,z}^{-1}e^{ik_{qp,z}(h+d)}\left(\vec{T}_{j,qp}\times\vec{k}_{qp}^{2}\right)\right)\cdot\vec{y}=0\,.$$

mivaqcioT axla yuradReba imas, rom amave sasazRvro pirobebs unda akmayofilebdnen ara mxolod jamuri velebi, aramed aseve calkeuli A_j

denis mier Seqmnili velebi. Tu gaviTvaliswinebT naTqvams da gamoviyenebT aseve $\vec{R}_{j,qp}$, $\vec{Q}_{j,qp}$, $\vec{S}_{j,qp}$, $\vec{T}_{j,qp}$ veqtorebis gamosaxulebebs, maSin ucnobi $B_{j,qp}$, $C_{j,qp}$, $D_{j,qp}$, $F_{j,qp}$, $G_{j,qp}$, $L_{j,qp}$, $\Gamma_{j,qp}$, $Y_{j,qp}$ koeficientebis mimarT miviRebT wrfiv gantolebaTa sistemas:

$$e^{-ik_{qp,z}h}\left(B_{j,qp}k_{p,x}+C_{j,qp}dx_j\right)-\left(k_{qp,z}/\varepsilon k'_{qp,z}\right)e^{ik'_{qp,z}h}\left(D_{j,qp}k_{p,x}+F_{j,qp}dx_j\right)-$$
$$-\left(k_{qp,z}/\varepsilon k'_{qp,z}\right)e^{-ik'_{qp,z}h}\left(G_{j,qp}k_{p,x}+L_{j,qp}dx_j\right)=-e^{ik_{qp,z}h}\left(\vec{P}^2_{j,qp}\cdot\vec{x}\right),$$

$$\mu\varepsilon k'_{qp,z}e^{-ik_{qp,z}h}\left(2B_{j,qp}k_{q,y}+C_{j,qp}dy_j\right)-$$
$$-e^{ik'_{qp,z}h}\left(D_{j,qp}k_{q,y}\left(k_{qp,z}-k'_{qp,z}\right)-F_{j,qp}k'_{qp,z}dy_j\right)-$$
$$-e^{-ik'_{qp,z}h}\left(G_{j,qp}k_{q,y}\left(k_{qp,z}+k'_{qp,z}\right)+L_{j,qp}k'_{qp,z}dy_j\right)=-k^2\mu\varepsilon k'_{qp,z}e^{ik_{qp,z}h}dy_j,$$

$$e^{-ik_{qp,z}h}\left(B_{j,qp}k_{q,y}+C_{j,qp}dy_j\right)-\left(k_{qp,z}/\varepsilon k'_{qp,z}\right)e^{ik'_{qp,z}h}\left(D_{j,qp}k_{q,y}+F_{j,qp}dy_j\right)-$$
$$-\left(k_{qp,z}/\varepsilon k'_{qp,z}\right)e^{-ik'_{qp,z}h}\left(G_{j,qp}k_{q,y}+L_{j,qp}dy_j\right)=-e^{ik_{qp,z}h}\left(\vec{P}^2_{j,qp}\cdot\vec{y}\right),$$

$$\mu\varepsilon k'_{qp,z}e^{-ik_{qp,z}h}\left(2B_{j,qp}k_{p,x}+C_{j,qp}dx_j\right)-$$
$$-e^{ik'_{qp,z}h}\left(D_{j,qp}k_{p,x}\left(k_{qp,z}-k'_{qp,z}\right)-F_{j,qp}k'_{qp,z}dx_j\right)-$$
$$-e^{-ik'_{qp,z}h}\left(G_{j,qp}k_{p,x}\left(k_{qp,z}+k'_{qp,z}\right)+L_{j,qp}k'_{qp,z}dx_j\right)=-k^2\mu\varepsilon k'_{qp,z}e^{ik_{qp,z}h}dx_j$$

$$e^{ik'_{qp,z}(h+d)}\left(D_{j,qp}k_{p,x}+F_{j,qp}dx_j\right)+$$
$$+e^{-ik'_{qp,z}(h+d)}\left(G_{j,qp}k_{p,x}+L_{j,qp}dx_j\right)-\left(\varepsilon k'_{qp,z}/k_{qp,z}\right)e^{ik_{qp,z}(h+d)}\left(\Gamma_{j,qp}k_{p,x}+Y_{j,qp}dx_j\right)=0,$$

$$e^{ik'_{qp,z}(h+d)}\left(D_{j,qp}k_{q,y}\left(k_{qp,z}-k'_{qp,z}\right)-F_{j,qp}k'_{qp,z}dy_j\right)+$$
$$+e^{-ik'_{qp,z}(h+d)}\left(G_{j,qp}k_{q,y}\left(k_{qp,z}+k'_{qp,z}\right)+L_{j,qp}k'_{qp,z}dy_j\right)=-\mu\varepsilon k'_{qp,z}e^{ik_{qp,z}(h+d)}Y_{j,qp}dy_j$$

$$e^{ik'_{qp,z}(h+d)}\left(D_{j,qp}k_{q,y}+F_{j,qp}dy_j\right)+$$
$$+e^{-ik'_{qp,z}(h+d)}\left(G_{j,qp}k_{q,y}+L_{j,qp}dy_j\right)-\left(\varepsilon k'_{qp,z}/k_{qp,z}\right)e^{ik_{qp,z}(h+d)}\left(\Gamma_{j,qp}k_{q,y}+Y_{j,qp}dy_j\right)=0$$

$$e^{ik'_{qp,z}(h+d)}\left(D_{j,qp}k_{p,x}\left(k_{qp,z}-k'_{qp,z}\right)-F_{j,qp}k'_{qp,z}dx_j\right)+$$
$$+e^{-ik'_{qp,z}(h+d)}\left(G_{j,qp}k_{p,x}\left(k_{qp,z}+k'_{qp,z}\right)+L_{j,qp}k'_{qp,z}dx_j\right)=-\mu\varepsilon k'_{qp,z}e^{ik_{qp,z}(h+d)}Y_{j,qp}dx_j.$$

am sistemis pirdapiri amoxsna garkveul sirTules warmoadgens. amitom ufro martivia misi amoxsna kompiuteruli modelirebis saSualebiT.

amoxsnis Semdeg ucnobi rCeba mxolod A_j denis amplitudebi mesris elementSi da maTi gansazRvra (3.2.6) sasazRvro pirobidan SeiZleba. elementi warmodgenilia rogorc segmentebis erToblioba da es sasazRvro piroba yovel aseT segmentze unda srulდebodes:

$$\left(\vec{E}_{inc}\left(\vec{r}_\sigma\right)+\vec{E}_1\left(\vec{r}_\sigma\right)+\vec{E}_e\left(\vec{r}_\sigma\right)\right)\cdot d\vec{l}_\sigma=0,\ \ \sigma=1,2,...,N.$$

aq $\vec{r}_\sigma=\vec{r}_\sigma\left\{x_\sigma,y_\sigma,dr_0\right\}$ warmoadgens dl_σ segmentis zedapirze aRebuli wertilis radiusveqtori. es bolo gamosaxuleba aseve gantolebaTa sistemas warmoadgens da misi amoxsna kvlav kompiuteris saSualebiT SeiZleba.

amis Semdeg napovnia yvela ucnobi koeficienti da maTi CasmiT velebis gamosaxulebaSi, vpoulobT velebis mniSvnelobebs yvela areSi.

Semdeg ganixileba dasmuli amocanis amoxsnis sxva gza. amisaTvis mesridan dieleqtrikisaken mimavali $\vec{E}_2(\vec{r})$, $\vec{H}_2(\vec{r})$ velis magivrad ganixileba mxolod misi erTi harmonika da Seswavlilia am harmonikis urTierTqmedeba am dieleqtrikTan. garda amisa, am harmonikis polarizaciis veqtori daSlilia or mdgenelad. pirveli mdgeneli imyofeba dacemis sibrtyeSi, xolo meore – am sibrtyis marTobulia. Seswavlilia cal-calke am ori velis difraqcia dieleqtrikul fenaze da gabneuli velis amplitudebi kvlav sasazRvro pirobebidan ganisazRvreba.

polarizaciis veqtoris daSla paralelur da marTobul mdgenelebad. ganvixiloT mesridan dieleqtrikul fenisaken mimavali erTerTi harmonika:

$$\vec{E}_{j,mn}(\vec{r}) = (1/2\omega\varepsilon_0 d_1 d_2)\, k_{mn,z}^{-1} e^{i\vec{k}_{mn}^2\cdot(\vec{r}-\vec{r}_j)} \vec{P}_{j,mn}^2 ,$$

$$\vec{H}_{j,mn}(\vec{r}) = (1/2d_1 d_2)\, k_{mn,z}^{-1} e^{i\vec{k}_{mn}^2\cdot(\vec{r}-\vec{r}_j)} \left(d\vec{l}_j \times \vec{k}_{mn}^2\right) = (1/2d_1 d_2 k^2)\, k_{mn,z}^{-1} e^{i\vec{k}_{mn}^2\cdot(\vec{r}-\vec{r}_j)} \left(\vec{k}_{mn}^2 \times \vec{P}_{mn}^2\right),$$

sadac

$$\vec{P}_{j,mn}^2 = \vec{k}_{mn}^2 \times \left(\vec{k}_{mn}^2 \times d\vec{l}_j\right) = \vec{k}_{mn}^2 \left(\vec{k}_{mn}^2 \cdot d\vec{l}_j\right) - k^2 d\vec{l}_j \text{ -}$$

am harmonikis polarizaciis veqtoria. gasagebia, rom sruli veli aris aseTi harmonikebis jami:

$$\vec{E}_2(\vec{r}) = \sum_{j=1}^{N} A_j \sum_{m=-\infty}^{+\infty} \sum_{n=-\infty}^{+\infty} \vec{E}_{j,mn}(\vec{r}), \quad \vec{H}_2(\vec{r}) = \sum_{j=1}^{N} A_j \sum_{m=-\infty}^{+\infty} \sum_{n=-\infty}^{+\infty} \vec{H}_{j,mn}(\vec{r}).$$

warmovidginoT axla $\vec{P}_{j,mn}^2$ veqtori, rogorc $\vec{P}_{j,mn}^2 = \vec{P}_{j,mn,\parallel}^2 + \vec{P}_{j,mn,\perp}^2$, sadac $\vec{P}_{j,mn,\parallel}^2$ imyofeba imave sibrtyeSi, romelSicaa $\vec{k}_{mn}^2$ da $\vec{z}$ ortveqtori (dacemis sibrtye), xolo $\vec{P}_{j,mn,\perp}^2$ am sibrtyis marTobulia.

naTqvamis Tanaxmad es veqtorebi unda akmayofilebdnen Semdeg pirobebs:

$$\left(\vec{P}_{j,mn,\parallel}^2 \times \vec{k}_{mn}^2\right)\cdot \vec{z} = 0, \quad \vec{P}_{j,mn,\perp}^2 \cdot \vec{k}_{mn}^2 = 0, \quad \vec{P}_{j,mn,\perp}^2 \cdot \vec{z} = 0 .$$

gavSaloT $\vec{P}_{j,mn,\parallel}^2$ da $\vec{P}_{j,mn,\perp}^2$ veqtorebi $\vec{x},\vec{y},\vec{z}$ bazisSi:

$$\vec{P}_{j,mn,\parallel}^2 = a_{j,mn,x}\vec{x} + a_{j,mn,y}\vec{y} + a_{j,mn,z}\vec{z}, \quad \vec{P}_{j,mn,\perp}^2 = b_{j,mn,x}\vec{x} + b_{j,mn,y}\vec{y} + b_{j,mn,z}\vec{z} .$$

maSin moyvanil pirobebidan, gaSlis ucnob koeficientebis mimarT vRebulobT

$$a_{j,mn,x}k_{m,y} - a_{j,mn,y}k_{n,x} = 0, \quad b_{j,mn,x}k_{n,x} + b_{j,mn,y}k_{m,y} = 0, \quad b_{j,mn,z} = 0$$

da aqedan

$$a_{j,mn,y} = \left(k_{m,y}/k_{n,x}\right) a_{j,mn,x}, \quad b_{j,mn,y} = -\left(k_{n,x}/k_{m,y}\right) b_{j,mn,x}, \quad b_{j,mn,z} = 0 .$$

maSasadame,

$$\vec{P}_{j,mn,\parallel}^2 = a_{j,mn,x}\vec{x} + \left(k_{m,y}/k_{n,x}\right) a_{j,mn,x}\vec{y} + a_{j,mn,z}\vec{z}, \quad \vec{P}_{j,mn,\perp}^2 = b_{j,mn,x}\vec{x} - \left(k_{n,x}/k_{m,y}\right) b_{j,mn,x}\vec{y},$$

saidanac

$$\vec{P}_{j,mn}^2 = \left(a_{j,mn,x} + b_{j,mn,x}\right)\vec{x} + \left(\left(k_{m,y}/k_{n,x}\right) a_{j,mn,x} - \left(k_{n,x}/k_{m,y}\right) b_{j,mn,x}\right)\vec{y} + a_{j,mn,z}\vec{z} .$$

magram, $\vec{P}_{j,mn}^2$ veqtoris gamosaxulebidan gamomdinareobs, rom

$$\vec{P}_{j,mn}^2 = \left(k_{n,x}\left(\vec{k}_{mn}^2 \cdot d\vec{l}_j\right) - k^2 dx_j\right)\vec{x} + \left(k_{m,y}\left(\vec{k}_{mn}^2 \cdot d\vec{l}_j\right) - k^2 dy_j\right)\vec{y} - k_{mn,z}\left(\vec{k}_{mn}^2 \cdot d\vec{l}_j\right)\vec{z}$$

da amitom unda srulebodes Semdegi pirobebi

$$\begin{cases} a_{j,mn,x}+b_{j,mn,x}=k_{n,x}\left(\vec{k}_{mn}^{2}\cdot d\vec{l}_{j}\right)-k^{2}dx_{j} \\ \left(k_{m,y}/k_{n,x}\right)a_{j,mn,x}-\left(k_{n,x}/k_{m,y}\right)b_{j,mn,x}=k_{m,y}\left(\vec{k}_{mn}^{2}\cdot d\vec{l}_{j}\right)-k^{2}dy_{j} \\ a_{j,mn,z}=-k_{mn,z}\left(\vec{k}_{mn}^{2}\cdot d\vec{l}_{j}\right). \end{cases}$$

am sistemis amoxsnis Sedegad viciT ukve yvela koeficientebi:

$$a_{j,mn,x}=k_{n,x}\left(1-k^{2}\left(k_{n,x}^{2}+k_{m,y}^{2}\right)^{-1}\right)\left(\vec{k}_{mn}^{2}\cdot d\vec{l}_{j}\right),\ \ a_{j,mn,y}=k_{m,y}\left(1-k^{2}\left(k_{n,x}^{2}+k_{m,y}^{2}\right)^{-1}\right)\left(\vec{k}_{mn}^{2}\cdot d\vec{l}_{j}\right),$$

$$a_{j,mn,z}=-k_{mn,z}\left(\vec{k}_{mn}^{2}\cdot d\vec{l}_{j}\right),\ \ b_{j,mn,x}=k^{2}\left(k_{n,x}\left(k_{n,x}^{2}+k_{m,y}^{2}\right)^{-1}\left(\vec{k}_{mn}^{2}\cdot d\vec{l}_{j}\right)-dx_{j}\right),$$

$$b_{j,mn,y}=-k^{2}\left(k_{n,x}/k_{m,y}\right)\left(k_{n,x}\left(k_{n,x}^{2}+k_{m,y}^{2}\right)^{-1}\left(\vec{k}_{mn}^{2}\cdot d\vec{l}_{j}\right)-dx_{j}\right),\ \ b_{j,mn,z}=0.$$

aqedan gamomdinare,

$$\vec{P}_{j,mn,\parallel}^{2}=\vec{k}_{mn}^{2}\left(\vec{k}_{mn}^{2}\cdot d\vec{l}_{j}\right)-k^{2}\left(k_{n,x}^{2}+k_{m,y}^{2}\right)^{-1}\left(\vec{k}_{mn}^{2}\cdot d\vec{l}_{j}\right)\left(k_{n,x}\vec{x}+k_{m,y}\vec{y}\right)$$

$$\vec{P}_{j,mn,\perp}^{2}=k^{2}\left(k_{n,x}\left(k_{n,x}^{2}+k_{m,y}^{2}\right)^{-1}\left(\vec{k}_{mn}^{2}\cdot d\vec{l}_{j}\right)-dx_{j}\right)\left(\vec{x}-\left(k_{n,x}/k_{m,y}\right)\vec{y}\right).$$

dasmulli amocanis amoxsnis meore meTodi. Cven unda amovxsnaT difraqciis amocana calkeul harmonikisaTvis. amisaTvis dacemuli harmonikis gamosaxulebaSi $\vec{P}_{j,mn}^{2}$ veqtori warmovidginoT zeviT moyvanili jamis saxiT. miviRebT:

$$\vec{E}_{j,mn}\left(\vec{r}\right)=\vec{E}_{j,mn,\parallel}^{0}e^{i\vec{k}_{mn}^{2}\cdot\left(\vec{r}-\vec{r}_{j}\right)}+\vec{E}_{j,mn,\perp}^{0}e^{i\vec{k}_{mn}^{2}\cdot\left(\vec{r}-\vec{r}_{j}\right)},\ \ \vec{H}_{j,mn}\left(\vec{r}\right)=\vec{H}_{j,mn,\perp}^{0}e^{i\vec{k}_{mn}^{2}\cdot\left(\vec{r}-\vec{r}_{j}\right)}+\vec{H}_{j,mn,\parallel}^{0}e^{i\vec{k}_{mn}^{2}\cdot\left(\vec{r}-\vec{r}_{j}\right)},$$

sadac

$$\vec{E}_{j,mn,\parallel}^{0}=\left(1/2\omega\varepsilon_{0}d_{1}d_{2}\right)k_{mn,z}^{-1}\vec{P}_{j,mn,\parallel}^{2},\ \ \vec{E}_{j,mn,\perp}^{0}=\left(1/2\omega\varepsilon_{0}d_{1}d_{2}\right)k_{mn,z}^{-1}\vec{P}_{j,mn,\perp}^{2},$$

$$\vec{H}_{j,mn,\perp}^{0}=\left(1/2d_{1}d_{2}\,k^{2}\right)k_{mn,z}^{-1}\left(\vec{k}_{mn}^{2}\times\vec{P}_{j,mn,\parallel}^{2}\right),\ \ \vec{H}_{j,mn,\parallel}^{0}=\left(1/2d_{1}d_{2}\,k^{2}\right)k_{mn,z}^{-1}\left(\vec{k}_{mn}^{2}\times\vec{P}_{j,mn,\perp}^{2}\right).$$

es Canaweri gvaZlevs saSualebas davweroT analogiuri gamosaxulebebi danarCen ucnob difragirebul velebisaTvis. marTlac,

1. arekvlil vels unda gaaCndes Semdegi saxe:

$$\vec{E}_{j,mn}^{e}\left(\vec{r}\right)=\vec{E}_{j,mn,\parallel}^{0,e}e^{i\vec{k}_{mn}^{1}\cdot\left(\vec{r}-\vec{r}_{j}\right)}+\vec{E}_{j,mn,\perp}^{0,e}e^{i\vec{k}_{mn}^{1}\cdot\left(\vec{r}-\vec{r}_{j}\right)},\ \ \vec{H}_{j,mn}^{e}\left(\vec{r}\right)=\vec{H}_{j,mn,\perp}^{0,e}e^{i\vec{k}_{mn}^{1}\cdot\left(\vec{r}-\vec{r}_{j}\right)}+\vec{H}_{j,mn,\parallel}^{0,e}e^{i\vec{k}_{mn}^{1}\cdot\left(\vec{r}-\vec{r}_{j}\right)}$$

$$\vec{E}_{j,mn,\parallel}^{0,e}=R_{j,mn,\parallel}\left(1/2\omega\varepsilon_{0}d_{1}d_{2}\right)k_{mn,z}^{-1}\vec{P}_{j,mn,\parallel}^{1},\ \ \vec{E}_{j,mn,\perp}^{0,e}=R_{j,mn,\perp}\left(1/2\omega\varepsilon_{0}d_{1}d_{2}\right)k_{mn,z}^{-1}\vec{P}_{j,mn,\perp}^{1},$$

$$\vec{H}_{j,mn,\perp}^{0,e}=R_{j,mn,\parallel}\left(1/2d_{1}d_{2}\,k^{2}\right)k_{mn,z}^{-1}\left(\vec{k}_{mn}^{1}\times\vec{P}_{j,mn,\parallel}^{1}\right),\ \ \vec{H}_{j,mn,\parallel}^{0,e}=R_{j,mn,\perp}\left(1/2d_{1}d_{2}\,k^{2}\right)k_{mn,z}^{-1}\left(\vec{k}_{mn}^{1}\times\vec{P}_{j,mn,\perp}^{1}\right),$$

sadac analogiurad

$$\vec{P}_{j,mn,\parallel}^{1}=\vec{k}_{mn}^{1}\left(\vec{k}_{mn}^{1}\cdot d\vec{l}_{j}\right)-k^{2}\left(k_{n,x}^{2}+k_{m,y}^{2}\right)^{-1}\left(\vec{k}_{mn}^{1}\cdot d\vec{l}_{j}\right)\left(k_{n,x}\vec{x}+k_{m,y}\vec{y}\right)$$

$$\vec{P}_{j,mn,\perp}^{1}=k^{2}\left(k_{n,x}\left(k_{n,x}^{2}+k_{m,y}^{2}\right)^{-1}\left(\vec{k}_{mn}^{1}\cdot d\vec{l}_{j}\right)-dx_{j}\right)\left(\vec{x}-\left(k_{n,x}/k_{m,y}\right)\vec{y}\right),$$

xolo $R_{j,mn,\parallel}$ da $R_{j,mn,\perp}$ arekvlis ucnobi koeficientebia.

2. gardatexili velisTvis gveqneba

$$\vec{E}_{j,mn}^{f}\left(\vec{r}\right)=\vec{E}_{j,mn,\parallel}^{0,f}e^{i\vec{k}_{mn}^{f}\cdot\left(\vec{r}-\vec{r}_{j}\right)}+\vec{E}_{j,mn,\perp}^{0,f}e^{i\vec{k}_{mn}^{f}\cdot\left(\vec{r}-\vec{r}_{j}\right)},\ \ \vec{H}_{j,mn}^{f}\left(\vec{r}\right)=\vec{H}_{j,mn,\perp}^{0,f}e^{i\vec{k}_{mn}^{f}\cdot\left(\vec{r}-\vec{r}_{j}\right)}+\vec{H}_{j,mn,\parallel}^{0,f}e^{i\vec{k}_{mn}^{f}\cdot\left(\vec{r}-\vec{r}_{j}\right)},$$

$$\vec{E}_{j,mn,\parallel}^{0,f}=A_{j,mn,\parallel}\left(1/2\omega\varepsilon_{0}\varepsilon d_{1}d_{2}\right)k_{mn,z}^{\prime-1}\vec{P}_{j,mn,\parallel}^{f},\ \ \vec{E}_{j,mn,\perp}^{0,f}=A_{j,mn,\perp}\left(1/2\omega\varepsilon_{0}\varepsilon d_{1}d_{2}\right)k_{mn,z}^{\prime-1}\vec{P}_{j,mn,\perp}^{f},$$

$$\vec H^{0,f}_{j,mn,\perp} = A_{j,mn,\|}\left(1/2d_1d_2\,\varepsilon\mu k^2\right)k'^{-1}_{mn,z}\left(\vec k^f_{mn}\times\vec P^f_{j,mn,\|}\right),\ \vec H^{0,f}_{j,mn,\|} = A_{j,mn,\perp}\left(1/2d_1d_2\,\varepsilon\mu k^2\right)k'^{-1}_{mn,z}\left(\vec k^f_{mn}\times\vec P^f_{j,mn,\perp}\right),$$

$$\vec P^f_{j,mn,\|} = \vec k^f_{mn}\left(\vec k^f_{mn}\cdot d\vec l_j\right)-\varepsilon\mu k^2\left(k^2_{n,x}+k^2_{m,y}\right)^{-1}\left(\vec k^f_{mn}\cdot d\vec l_j\right)\left(k_{n,x}\vec x+k_{m,y}\vec y\right),$$

$$\vec P^f_{j,mn,\perp} = \varepsilon\mu k^2\left(k_{n,x}\left(k^2_{n,x}+k^2_{m,y}\right)^{-1}\left(\vec k^f_{mn}\cdot d\vec l_j\right)-dx_j\right)\left(\vec x-\left(k_{n,x}/k_{m,y}\right)\vec y\right),$$

sadac $A_{j,mn,\|}$ da $A_{j,mn,\perp}$ gardatexis ucnobi koeficientebia.

3. fenis SigniT arekvlili veli:

$$\vec E^g_{j,mn}(\vec r) = \vec E^{0,g}_{j,mn,\|}e^{i\vec k^g_{mn}\cdot(\vec r-\vec r_j)}+\vec E^{0,g}_{j,mn,\perp}e^{i\vec k^g_{mn}\cdot(\vec r-\vec r_j)},\ \ \vec H^g_{j,mn}(\vec r) = \vec H^{0,g}_{j,mn,\perp}e^{i\vec k^g_{mn}\cdot(\vec r-\vec r_j)}+\vec H^{0,g}_{j,mn,\|}e^{i\vec k^g_{mn}\cdot(\vec r-\vec r_j)},$$

$$\vec E^{0,g}_{j,mn,\|} = B_{j,mn,\|}\left(1/2\omega\varepsilon_0\varepsilon d_1d_2\right)k'^{-1}_{mn,z}\vec P^g_{j,mn,\|},\ \ \vec E^{0,g}_{j,mn,\perp} = B_{j,mn,\perp}\left(1/2\omega\varepsilon_0\varepsilon d_1d_2\right)k'^{-1}_{mn,z}\vec P^g_{j,mn,\perp},$$

$$\vec H^{0,g}_{j,mn,\perp} = B_{j,mn,\|}\left(1/2d_1d_2\,\varepsilon\mu k^2\right)k'^{-1}_{mn,z}\left(\vec k^g_{mn}\times\vec P^g_{j,mn,\|}\right),\ \vec H^{0,g}_{j,mn,\|} = B_{j,mn,\perp}\left(1/2d_1d_2\,\varepsilon\mu k^2\right)k'^{-1}_{mn,z}\left(\vec k^g_{mn}\times\vec P^g_{j,mn,\perp}\right),$$

$$\vec P^g_{j,mn,\|} = \vec k^g_{mn}\left(\vec k^g_{mn}\cdot d\vec l_j\right)-\varepsilon\mu k^2\left(k^2_{n,x}+k^2_{m,y}\right)^{-1}\left(\vec k^g_{mn}\cdot d\vec l_j\right)\left(k_{n,x}\vec x+k_{m,y}\vec y\right),$$

$$\vec P^g_{j,mn,\perp} = \varepsilon\mu k^2\left(k_{n,x}\left(k^2_{n,x}+k^2_{m,y}\right)^{-1}\left(\vec k^g_{mn}\cdot d\vec l_j\right)-dx_j\right)\left(\vec x-\left(k_{n,x}/k_{m,y}\right)\vec y\right),$$

sadac $B_{j,mn,\|}$ da $B_{j,mn,\perp}$ Sida arekvlis ucnobi koeficientebia.

4. gasuli veli:

$$\vec E^s_{j,mn}(\vec r) = \vec E^{0,s}_{j,mn,\|}e^{i\vec k^2_{mn}\cdot(\vec r-\vec r_j)}+\vec E^{0,s}_{j,mn,\perp}e^{i\vec k^2_{mn}\cdot(\vec r-\vec r_j)},\ \ \vec H^s_{j,mn}(\vec r) = \vec H^{0,s}_{j,mn,\perp}e^{i\vec k^2_{mn}\cdot(\vec r-\vec r_j)}+\vec H^{0,s}_{j,mn,\|}e^{i\vec k^2_{mn}\cdot(\vec r-\vec r_j)},$$

$$\vec E^{0,s}_{j,mn,\|} = T_{j,mn,\|}\left(1/2\omega\varepsilon_0 d_1d_2\right)k^{-1}_{mn,z}\vec P^2_{j,mn,\|},\ \ \vec E^{0,s}_{j,mn,\perp} = T_{j,mn,\perp}\left(1/2\omega\varepsilon_0 d_1d_2\right)k^{-1}_{mn,z}\vec P^2_{j,mn,\perp},$$

$$\vec H^{0,s}_{j,mn,\perp} = T_{j,mn,\|}\left(1/2d_1d_2\,k^2\right)k^{-1}_{mn,z}\left(\vec k^2_{mn}\times\vec P^2_{j,mn,\|}\right),\ \ \vec H^{0,s}_{j,mn,\|} = T_{j,mn,\perp}\left(1/2d_1d_2\,k^2\right)k^{-1}_{mn,z}\left(\vec k^2_{mn}\times\vec P^2_{j,mn,\perp}\right),$$

sadac $T_{j,mn,\|}$ da $T_{j,mn,\perp}$ ucnobi koeficientebia.

maSasadame, gagvaCnia ori damoukidebeli talRa: pirveli talRa: $\vec E^0_{j,mn,\|}e^{i\vec k^2_{mn}\cdot(\vec r-\vec r_j)}$, $\vec H^0_{j,mn,\perp}e^{i\vec k^2_{mn}\cdot(\vec r-\vec r_j)}$, romelic polarizebulia dacemis sibrtyeSi da aseve meore talRa: $\vec E^0_{j,mn,\perp}e^{i\vec k^2_{mn}\cdot(\vec r-\vec r_j)}$, $\vec H^0_{j,mn,\|}e^{i\vec k^2_{mn}\cdot(\vec r-\vec r_j)}$, romelic dacemis sibrtyis marTobuladaa polarizebuli. cal-calke SeviswavloT am talRebis urTierTqmedeba dieleqtrikul fenasTan.

dacemis sibrtyeSi polarizebuli talRa. am talRis difraqciis dros, dieleqtrikis zedapirebze unda sruldebodnen Semdegi sasazRvro pirobebi:

$$\begin{cases}\left.\left(E^0_{j,mn,\|}e^{i\vec k^2_{mn}\cdot(\vec r-\vec r_j)}-E^{0,e}_{j,mn,\|}e^{i\vec k^1_{mn}\cdot(\vec r-\vec r_j)}\right)\right|_{z=-h}\cos\vartheta = \left.\left(E^{0,f}_{j,mn,\|}e^{i\vec k^f_{mn}\cdot(\vec r-\vec r_j)}-E^{0,g}_{j,mn,\|}e^{i\vec k^g_{mn}\cdot(\vec r-\vec r_j)}\right)\right|_{z=-h}\cos\psi\\ \left.\left(E^{0,f}_{j,mn,\|}e^{i\vec k^f_{mn}\cdot(\vec r-\vec r_j)}-E^{0,g}_{j,mn,\|}e^{i\vec k^g_{mn}\cdot(\vec r-\vec r_j)}\right)\right|_{z=-(h+l)}\cos\psi = \left.E^{0,s}_{j,mn,\|}e^{i\vec k^2_{mn}\cdot(\vec r-\vec r_j)}\right|_{z=-(h+l)}\cos\vartheta\\ \left.\left(H^0_{j,mn,\perp}e^{i\vec k^2_{mn}\cdot(\vec r-\vec r_j)}+H^{0,e}_{j,mn,\perp}e^{i\vec k^1_{mn}\cdot(\vec r-\vec r_j)}\right)\right|_{z=-h} = \left.\left(H^{0,f}_{j,mn,\perp}e^{i\vec k^f_{mn}\cdot(\vec r-\vec r_j)}+H^{0,g}_{j,mn,\perp}e^{i\vec k^g_{mn}\cdot(\vec r-\vec r_j)}\right)\right|_{z=-h}\\ \left.\left(H^{0,f}_{j,mn,\perp}e^{i\vec k^f_{mn}\cdot(\vec r-\vec r_j)}+H^{0,g}_{j,mn,\perp}e^{i\vec k^g_{mn}\cdot(\vec r-\vec r_j)}\right)\right|_{z=-(h+l)} = \left.H^{0,s}_{j,mn,\perp}e^{i\vec k^2_{mn}\cdot(\vec r-\vec r_j)}\right|_{z=-(h+l)}.\end{cases}$$

moyvanili sistema garkveuli gamartivebis Semdeg Caiwereba rogorc

$$\begin{cases}\left(E^{0}_{j,mn,\parallel}e^{ik_{mn,z}h}-E^{0,e}_{j,mn,\parallel}e^{-ik_{mn,z}h}\right)\cos\vartheta=\left(E^{0,f}_{j,mn,\parallel}e^{ik'_{mn,z}h}-E^{0,g}_{j,mn,\parallel}e^{-ik'_{mn,z}h}\right)\cos\psi\\ \left(E^{0,f}_{j,mn,\parallel}e^{ik'_{mn,z}(h+l)}-E^{0,g}_{j,mn,\parallel}e^{-ik'_{mn,z}(h+l)}\right)\cos\psi=E^{0,s}_{j,mn,\parallel}e^{ik_{mn,z}(h+l)}\cos\vartheta\\ H^{0}_{j,mn,\perp}e^{ik_{mn,z}h}+H^{0,e}_{j,mn,\perp}e^{-ik_{mn,z}h}=H^{0,f}_{j,mn,\perp}e^{ik'_{mn,z}h}+H^{0,g}_{j,mn,\perp}e^{-ik'_{mn,z}h}\\ H^{0,f}_{j,mn,\perp}e^{ik'_{mn,z}(h+l)}+H^{0,g}_{j,mn,\perp}e^{-ik'_{mn,z}(h+l)}=H^{0,s}_{j,mn,\perp}e^{ik_{mn,z}(h+l)}.\end{cases}$$

aq ϑ da ψ Sesabamisad dacemis da gardatexis kuTxeebia. SeiZleba naCveneb iqnas, rom am gamosaxulebebidan gamomdinareobs Semdegi damokidebulebebi velis amplitudebs Soris:

$$H^{0}_{j,mn,\perp}=E^{0}_{j,mn,\parallel}/Z,\ \ H^{0,e}_{j,mn,\perp}=E^{0,e}_{j,mn,\parallel}/Z,\ \ H^{0,f}_{j,mn,\perp}=E^{0,f}_{j,mn,\parallel}/Z',$$

$$H^{0,g}_{j,mn,\perp}=E^{0,g}_{j,mn,\parallel}/Z',\ \ H^{0,s}_{j,mn,\perp}=E^{0,s}_{j,mn,\parallel}/Z,$$

sadac $Z=\sqrt{\mu_0/\varepsilon_0}$, $Z'=\sqrt{\mu_0\mu/\varepsilon_0\varepsilon}$ Tavisufali sivrcis da dieleqtrikis talRuri winaRobebia. amitom sistema sabolood Caiwereba rogorc

$$\begin{cases}E^{0}_{j,mn,\parallel}e^{ik_{mn,z}h}-E^{0,e}_{j,mn,\parallel}e^{-ik_{mn,z}h}=\left(\cos\psi/\cos\vartheta\right)\left(E^{0,f}_{j,mn,\parallel}e^{ik'_{mn,z}h}-E^{0,g}_{j,mn,\parallel}e^{-ik'_{mn,z}h}\right)\\ E^{0,s}_{j,mn,\parallel}e^{ik_{mn,z}(h+l)}=\left(\cos\psi/\cos\vartheta\right)\left(E^{0,f}_{j,mn,\parallel}e^{ik'_{mn,z}(h+l)}-E^{0,g}_{j,mn,\parallel}e^{-ik'_{mn,z}(h+l)}\right)\\ E^{0}_{j,mn,\parallel}e^{ik_{mn,z}h}+E^{0,e}_{j,mn,\parallel}e^{-ik_{mn,z}h}=\left(Z/Z'\right)\left(E^{0,f}_{j,mn,\parallel}e^{ik'_{mn,z}h}+E^{0,g}_{j,mn,\parallel}e^{-ik'_{mn,z}h}\right)\\ E^{0,s}_{j,mn,\parallel}e^{ik_{mn,z}(h+l)}=\left(Z/Z'\right)\left(E^{0,f}_{j,mn,\parallel}e^{ik'_{mn,z}(h+l)}+E^{0,g}_{j,mn,\parallel}e^{-ik'_{mn,z}(h+l)}\right).\end{cases}$$

Tu SemoviRebT aRniSvnebs

$$\tilde{Z}'=Z'\cos\psi+Z\cos\vartheta,\ \ \tilde{Z}=Z'\cos\psi-Z\cos\vartheta,$$

maSin am sistemis amonaxsni iqneba

$$E^{0,e}_{j,mn,\parallel}=E^{0}_{j,mn,\parallel}\tilde{Z}'\tilde{Z}\left(\tilde{Z}'^{2}-\tilde{Z}^{2}e^{2ik'_{mn,z}l}\right)^{-1}\left(e^{2ik'_{mn,z}l}-1\right)e^{2ik_{mn,z}h},$$

$$E^{0,f}_{j,mn,\parallel}=E^{0}_{j,mn,\parallel}\left(\cos\vartheta/\cos\psi\right)\tilde{Z}'\left(\tilde{Z}'+\tilde{Z}\right)\left(\tilde{Z}'^{2}-\tilde{Z}^{2}e^{2ik'_{mn,z}l}\right)^{-1}e^{i\left(k_{mn,z}-k'_{mn,z}\right)h},$$

$$E^{0,g}_{j,mn,\parallel}=E^{0}_{j,mn,\parallel}\left(\cos\vartheta/\cos\psi\right)\tilde{Z}\left(\tilde{Z}'+\tilde{Z}\right)\left(\tilde{Z}'^{2}-\tilde{Z}^{2}e^{2ik'_{mn,z}l}\right)^{-1}e^{i\left(k_{mn,z}+k'_{mn,z}\right)h+2ik'_{mn,z}l},$$

$$E^{0,s}_{j,mn,\parallel}=E^{0}_{j,mn,\parallel}\left(\tilde{Z}'^{2}-\tilde{Z}^{2}\right)\left(\tilde{Z}'^{2}-\tilde{Z}^{2}e^{2ik'_{mn,z}l}\right)^{-1}e^{i\left(k'_{mn,z}-k_{mn,z}\right)l}.$$

dacemis sibrtyis marTobulad polarizebuli talRa. misTvis, sasazRvro pirobebi gvaZlevs sxva gantolebaTa sistemas

$$\begin{cases}E^{0}_{j,mn,\perp}e^{ik_{mn,z}h}+E^{0,e}_{j,mn,\perp}e^{-ik_{mn,z}h}=E^{0,f}_{j,mn,\perp}e^{ik'_{mn,z}h}+E^{0,g}_{j,mn,\perp}e^{-ik'_{mn,z}h}\\ E^{0,f}_{j,mn,\perp}e^{ik'_{mn,z}(h+l)}+E^{0,g}_{j,mn,\perp}e^{-ik'_{mn,z}(h+l)}=E^{0,s}_{j,mn,\perp}e^{ik_{mn,z}(h+l)}\\ \left(H^{0}_{j,mn,\parallel}e^{ik_{mn,z}h}-H^{0,e}_{j,mn,\parallel}e^{-ik_{mn,z}h}\right)\cos\vartheta=\left(H^{0,f}_{j,mn,\parallel}e^{ik'_{mn,z}h}-H^{0,g}_{j,mn,\parallel}e^{-ik'_{mn,z}h}\right)\cos\psi\\ \left(H^{0,f}_{j,mn,\parallel}e^{ik'_{mn,z}(h+l)}-H^{0,g}_{j,mn,\parallel}e^{-ik'_{mn,z}(h+l)}\right)\cos\psi=H^{0,s}_{j,mn,\parallel}e^{ik_{mn,z}(h+l)}\cos\vartheta.\end{cases}$$

aqac SeiZleba naCveneb iqnas, rom magnituri da eleqtruli velis amplitudebi Sesabamisi garemos talRuri winaRobebiT gansxvavdebian:

$$H^{0}_{j,mn,\parallel}=E^{0}_{j,mn,\perp}/Z,\ \ H^{0,e}_{j,mn,\parallel}=E^{0,e}_{j,mn,\perp}/Z,\ \ H^{0,f}_{j,mn,\parallel}=E^{0,f}_{j,mn,\perp}/Z',$$

$$H^{0,g}_{j,mn,\parallel}=E^{0,g}_{j,mn,\perp}/Z',\ \ H^{0,s}_{j,mn,\parallel}=E^{0,s}_{j,mn,\perp}/Z,$$

$$Z=\sqrt{\mu_0/\varepsilon_0}\,,\quad Z'=\sqrt{\mu\mu_0/\varepsilon\varepsilon_0}\,.$$

amitom zemoT moyvanili sistema miiRebs Semdeg saboloo saxes:

$$\begin{cases} E^{0}_{j,mn,\perp}e^{ik_{mn,z}h}+E^{0,e}_{j,mn,\perp}e^{-ik_{mn,z}h}=E^{0,f}_{j,mn,\perp}e^{ik'_{mn,z}h}+E^{0,g}_{j,mn,\perp}e^{-ik'_{mn,z}h} \\ E^{0,s}_{j,mn,\perp}e^{ik_{mn,z}(h+l)}=E^{0,f}_{j,mn,\perp}e^{ik'_{mn,z}(h+l)}+E^{0,g}_{j,mn,\perp}e^{-ik'_{mn,z}(h+l)} \\ E^{0}_{j,mn,\perp}e^{ik_{mn,z}h}-E^{0,e}_{j,mn,\perp}e^{-ik_{mn,z}h}=\left(Z\cos\psi/Z'\cos\vartheta\right)\left(E^{0,f}_{j,mn,\perp}e^{ik'_{mn,z}h}-E^{0,g}_{j,mn,\perp}e^{-ik'_{mn,z}h}\right) \\ E^{0,s}_{j,mn,\perp}e^{ik_{mn,z}(h+l)}=\left(Z\cos\psi/Z'\cos\vartheta\right)\left(E^{0,f}_{j,mn,\perp}e^{ik'_{mn,z}(h+l)}-E^{0,g}_{j,mn,\perp}e^{-ik'_{mn,z}(h+l)}\right). \end{cases}$$

Tu SemoviRebT aRniSvnebs

$$\bar{Z}'=Z\cos\psi+Z'\cos\vartheta\,,\quad \bar{Z}=Z\cos\psi-Z'\cos\vartheta\,,$$

maSin am sistemis amonaxsns eqneba Semdegi saxe:

$$E^{0,e}_{j,mn,\perp}=E^{0}_{j,mn,\perp}\bar{Z}'\bar{Z}\left(\bar{Z}'^2-\bar{Z}^2e^{2ik'_{mn,z}l}\right)^{-1}\left(e^{2ik'_{mn,z}l}-1\right)e^{2ik_{mn,z}h},$$

$$E^{0,f}_{j,mn,\perp}=E^{0}_{j,mn,\perp}\bar{Z}'\left(\bar{Z}'-\bar{Z}\right)\left(\bar{Z}'^2-\bar{Z}^2e^{2ik'_{mn,z}l}\right)^{-1}e^{i\left(k_{mn,z}-k'_{mn,z}\right)h},$$

$$E^{0,g}_{j,mn,\perp}=E^{0}_{j,mn,\perp}\bar{Z}\left(\bar{Z}'-\bar{Z}\right)\left(\bar{Z}'^2-\bar{Z}^2e^{2ik'_{mn,z}l}\right)^{-1}e^{i\left(\left(k_{mn,z}+k'_{mn,z}\right)h+2k'_{mn,z}l\right)},$$

$$E^{0,s}_{j,mn,\perp}=E^{0}_{j,mn,\perp}\left(\bar{Z}'^2-\bar{Z}^2\right)\left(\bar{Z}'^2-\bar{Z}^2e^{2ik'_{mn,z}l}\right)^{-1}e^{i\left(k'_{mn,z}-k_{mn,z}\right)l}.$$

Tu gavaerTianebT am or miRebul amonaxsnebs, maSin gvecodineba ra velebi iqmneba erTi harmonikis difraqciis dros dieleqtrikul fenaze da aqedan advilad vipoviT jamur velebs. exla ucnobia mxolod denis amplitudebi mesris elementSi da maT Sesabamis sasazRvro pirobidan vipoviT.

damxmare gamomsxiveblebis meTodis gamoyeneba. wina paragrafSi Cven amovxseniT difraqciis amocana im SemTxvevisTvis, rodesac meseri imyofeboda dieleqtrikuli fenis SigniT da gamoviyeneT amisaTvis damxmare gamomsxivreblebis meTodi. es meTodi aseve SeiZleba gamoyenebul iqnas am SemTxvevaSic, anu rodesac meseri imyofeba aRniSnuli fenis gareT. amisaTvis fenis zedapirebis maxloblad unda avagoT oTxi damxmare zedapiri da ganvalagoT maTze damxmare gamomsxivreblebi.

ganixileba ori gare da ori Sida damxmare zedapirebi:

$$\left(x,y\right)\in\left(d_1\times d_2\right),\; z=-h+\delta,\; z=-\left(h+d\right)-\delta,\; z=-h-\delta,\; z=-\left(h+d\right)+\delta,$$

sadac δ warmoadgens manZils damxmare zedapirsa da fenis zedapiris Soris. aseve rogorc wina TavSi, Cven aqac unda ganvixiloT ori urTierTmarTobuli elementaruli wyaro, romlis velsac gaaCnia saxe (3.1.2) – (3.1.3):

$$\vec{G}_E\left(\vec{r},\vec{r}_\alpha\right)=\left(1/2\omega\varepsilon_0\varepsilon d_1d_2\right)\sum_{m=-\infty}^{+\infty}\sum_{n=-\infty}^{+\infty}e^{i\vec{k}_{\alpha,mn}\left(\vec{r}-\vec{r}_\alpha\right)}\left(\mu\varepsilon k^2-k^2_{n,x}-k^2_{m,y}\right)^{-1/2}\left(\vec{k}_{\alpha,mn}\left(\vec{k}_{\alpha,mn}\cdot\vec{p}_\alpha\right)-\mu\varepsilon k^2\vec{p}_\alpha\right),$$

$$\vec{G}_H\left(\vec{r},\vec{r}_\alpha\right)=\left(1/2d_1d_2\right)\sum_{n=-\infty}^{+\infty}\sum_{m=-\infty}^{+\infty}e^{i\vec{k}_{\alpha,mn}\left(\vec{r}-\vec{r}_\alpha\right)}\left(\mu\varepsilon k^2-k^2_{n,x}-k^2_{m,y}\right)^{-1/2}\left(\vec{p}_\alpha\times\vec{k}_{\alpha,mn}\right),$$

$$\vec{k}_{\alpha,mn}=\vec{k}_{\alpha,mn}\left\{k_{n,x},k_{m,y},\operatorname{sgn}\left(z-z_\alpha\right)\sqrt{\mu\varepsilon k^2-k^2_{n,x}-k^2_{m,y}}\right\},\quad k_{n,x}=k_x+2\pi n/d_1\,,\quad k_{m,y}=k_y+2\pi m/d_2\,.$$

maTi orientacia unda ganisazRvrebodes Sesabamisad $\vec{x}$ da $\vec{y}$ bazisuri veqtorebiT. amasTanave, orive elementarul wyaros damxmare gamomsxivebelSi unda gaaCndes sakuTari ucnobi amplitudebi. naTqvamis gaTvaliswinebiT, velebisTvis (I) – (IV) areebSi davwerT:

(I). aq gagvaCnia dacemuli da mesridan zeviT mimavali velebi:

$$\vec{E}_{inc}(\vec{r})+\vec{E}_1(\vec{r})=\vec{E}_0 e^{i\vec{k}\vec{r}}+\sum_{j=1}^{N} I_j \underset{\mu=\varepsilon=1}{\vec{G}_E}(\vec{r},\vec{r}_j),\quad \vec{H}_{inc}(\vec{r})+\vec{H}_1(\vec{r})=\vec{H}_0 e^{i\vec{k}\vec{r}}+\sum_{j=1}^{N} I_j \underset{\mu=\varepsilon=1}{\vec{G}_H}(\vec{r},\vec{r}_j),$$

$$z_j=0,\quad \vec{p}_j=d\vec{l}_j,\quad z>dr_0,\quad \operatorname{sgn}(z-z_j)=1.$$

(II). gagvaCnia mesridan qveviT mimavali veli da aseve veli romelic $z=-h-\delta$ damxmare zedapiriT aRiwereba:

$$\vec{E}_2(\vec{r})+\vec{E}_e(\vec{r})=\sum_{j=1}^{N} I_j \underset{\mu=\varepsilon=1}{\vec{G}_E}(\vec{r},\vec{r}_j)+\sum_{\alpha=1}^{Q}\sum_{\alpha'=1}^{P}\left(A_{\alpha\alpha'}\underset{\mu=\varepsilon=1}{\vec{G}_E^x}(\vec{r},\vec{r}_{\alpha\alpha'})+B_{\alpha\alpha'}\underset{\mu=\varepsilon=1}{\vec{G}_E^y}(\vec{r},\vec{r}_{\alpha\alpha'})\right),$$

$$\vec{H}_2(\vec{r})+\vec{H}_e(\vec{r})=\sum_{j=1}^{N} I_j \underset{\mu=\varepsilon=1}{\vec{G}_H}(\vec{r},\vec{r}_j)+\sum_{\alpha=1}^{Q}\sum_{\alpha'=1}^{P}\left(A_{\alpha\alpha'}\underset{\mu=\varepsilon=1}{\vec{G}_H^x}(\vec{r},\vec{r}_{\alpha\alpha'})+B_{\alpha\alpha'}\underset{\mu=\varepsilon=1}{\vec{G}_H^y}(\vec{r},\vec{r}_{\alpha\alpha'})\right),$$

$$z_j=0,\quad \vec{p}_j=d\vec{l}_j,\quad z_{\alpha\alpha'}=-h-\delta\quad -h<z<-dr_0,\quad \operatorname{sgn}(z-z_j)=-1,\quad \operatorname{sgn}(z-z_{\alpha\alpha'})=1.$$

aq $\alpha\alpha'$ indeqsebi miuRiTeben damxmare gamomsxiveblis nomers da maSasadame TiToeul damxmare zedapirze maTi raodenobaa $Q\times P$.

(III). aq gagvaCnia $z=-h+\delta$ da $z=-(h+d)-\delta$ gare zedapirebis mier Seqmnili velebi:

$$\vec{E}_f(\vec{r})+\vec{E}_g(\vec{r})=\sum_{\beta=1}^{Q}\sum_{\beta'=1}^{P}\left(C_{\beta\beta'}\vec{G}_E^x(\vec{r},\vec{r}_{\beta\beta'})+D_{\beta\beta'}\vec{G}_E^y(\vec{r},\vec{r}_{\beta\beta'})\right)+$$

$$+\sum_{\gamma=1}^{Q}\sum_{\gamma'=1}^{P}\left(F_{\gamma\gamma'}\vec{G}_E^x(\vec{r},\vec{r}_{\gamma\gamma'})+L_{\gamma\gamma'}\vec{G}_E^y(\vec{r},\vec{r}_{\gamma\gamma'})\right),$$

$$\vec{H}_f(\vec{r})+\vec{H}_g(\vec{r})=\sum_{\beta=1}^{Q}\sum_{\beta'=1}^{P}\left(C_{\beta\beta'}\vec{G}_H^x(\vec{r},\vec{r}_{\beta\beta'})+D_{\beta\beta'}\vec{G}_H^y(\vec{r},\vec{r}_{\beta\beta'})\right)+$$

$$+\sum_{\gamma=1}^{Q}\sum_{\gamma'=1}^{P}\left(F_{\gamma\gamma'}\vec{G}_H^x(\vec{r},\vec{r}_{\gamma\gamma'})+L_{\gamma\gamma'}\vec{G}_H^y(\vec{r},\vec{r}_{\gamma\gamma'})\right),$$

$$z_{\beta\beta'}=-h+\delta,\quad z_{\gamma\gamma'}=-(h+d)-\delta,\quad -(h+d)<z<-h,\quad \operatorname{sgn}(z-z_{\beta\beta'})=-1,\quad \operatorname{sgn}(z-z_{\gamma\gamma'})=1.$$

(IV). aq mxolod $z=-(h+d)+\delta$ damxmare zedapiridan qveviT mimavali (gasuli) veli gagvaCnia:

$$\vec{E}_s(\vec{r})=\sum_{\sigma=1}^{Q}\sum_{\sigma'=1}^{P}\left(K_{\sigma\sigma'}\underset{\mu=\varepsilon=1}{\vec{G}_E^x}(\vec{r},\vec{r}_{\sigma\sigma'})+R_{\sigma\sigma'}\underset{\mu=\varepsilon=1}{\vec{G}_E^y}(\vec{r},\vec{r}_{\sigma\sigma'})\right),$$

$$\vec{H}_s(\vec{r})=\sum_{\sigma=1}^{Q}\sum_{\sigma'=1}^{P}\left(K_{\sigma\sigma'}\underset{\mu=\varepsilon=1}{\vec{G}_H^x}(\vec{r},\vec{r}_{\sigma\sigma'})+R_{\sigma\sigma'}\underset{\mu=\varepsilon=1}{\vec{G}_H^y}(\vec{r},\vec{r}_{\sigma\sigma'})\right),$$

$$z_{\sigma\sigma'}=-(h+d)+\delta,\quad z<-(h+d),\quad \operatorname{sgn}(z-z_{\sigma\sigma'})=-1.$$

maSasadame, ucnobi velebi gamosaxulia periuduli grinis funqciebiT romelnic imyofebian damxmare zedapirebze da aseve mesris elementis gaswvriv.

Cveni amocana kvlav dayvanilia imaze rom vipovoT denis ucnobi I_j amplitudebi ($j=1,2,...,N$) da damxmare gamomsxivblebis amplitudebi $A_{\alpha\alpha'}$, $B_{\alpha\alpha'}$, $C_{\beta\beta'}$, $D_{\beta\beta'}$, $F_{\gamma\gamma'}$, $L_{\gamma\gamma'}$, $K_{\sigma\sigma'}$, $R_{\sigma\sigma'}$, sadac ($\alpha,\beta,\gamma,\sigma=1,2,...,Q$, $\alpha',\beta',\gamma',\sigma'=1,2,...,P$). sul gagvaCnia $8Q\times P+N$ ucnobi da isini (3.2.6), (3.2.7) sasazRvro pirobebidan unda vipovoT. dieleqtrikis zedapirze moviTxovT sasazRvro pirobis Sesrulebas $Q\times P$ raodenobis gansxvavebul wertilSi. aseve viTxovT sasazRvro pirobis Sesrulebas mesris elementis N segmentze:

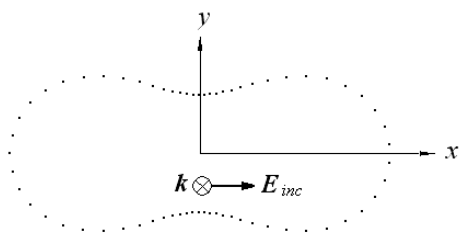
$$
\begin{cases}
\left(\vec{E}_2\left(\vec{r}_{\varphi\varphi'}\right)+\vec{E}_e\left(\vec{r}_{\varphi\varphi'}\right)\right)\Big|_{z_{\varphi\varphi'}=-h}\cdot\vec{x}=\left(\vec{E}_f\left(\vec{r}_{\varphi\varphi'}\right)+\vec{E}_g\left(\vec{r}_{\varphi\varphi'}\right)\right)\Big|_{z_{\varphi\varphi'}=-h}\cdot\vec{x} \\
\left(\vec{H}_2\left(\vec{r}_{\varphi\varphi'}\right)+\vec{H}_e\left(\vec{r}_{\varphi\varphi'}\right)\right)\Big|_{z_{\varphi\varphi'}=-h}\cdot\vec{x}=\left(\vec{H}_f\left(\vec{r}_{\varphi\varphi'}\right)+\vec{H}_g\left(\vec{r}_{\varphi\varphi'}\right)\right)\Big|_{z_{\varphi\varphi'}=-h}\cdot\vec{x} \\
\left(\vec{E}_2\left(\vec{r}_{\varphi\varphi'}\right)+\vec{E}_e\left(\vec{r}_{\varphi\varphi'}\right)\right)\Big|_{z_{\varphi\varphi'}=-h}\cdot\vec{y}=\left(\vec{E}_f\left(\vec{r}_{\varphi\varphi'}\right)+\vec{E}_g\left(\vec{r}_{\varphi\varphi'}\right)\right)\Big|_{z_{\varphi\varphi'}=-h}\cdot\vec{y} \\
\left(\vec{H}_2\left(\vec{r}_{\varphi\varphi'}\right)+\vec{H}_e\left(\vec{r}_{\varphi\varphi'}\right)\right)\Big|_{z_{\varphi\varphi'}=-h}\cdot\vec{y}=\left(\vec{H}_f\left(\vec{r}_{\varphi\varphi'}\right)+\vec{H}_g\left(\vec{r}_{\varphi\varphi'}\right)\right)\Big|_{z_{\varphi\varphi'}=-h}\cdot\vec{y} \\
\left(\vec{E}_f\left(\vec{r}_{\varphi\varphi'}\right)+\vec{E}_g\left(\vec{r}_{\varphi\varphi'}\right)\right)\Big|_{z_{\varphi\varphi'}=-(h+d)}\cdot\vec{x}=\vec{E}_s\left(\vec{r}_{\varphi\varphi'}\right)\Big|_{z_{\varphi\varphi'}=-(h+d)}\cdot\vec{x} \\
\left(\vec{H}_f\left(\vec{r}_{\varphi\varphi'}\right)+\vec{H}_g\left(\vec{r}_{\varphi\varphi'}\right)\right)\Big|_{z_{\varphi\varphi'}=-(h+d)}\cdot\vec{x}=\vec{H}_s\left(\vec{r}_{\varphi\varphi'}\right)\Big|_{z_{\varphi\varphi'}=-(h+d)}\cdot\vec{x} \\
\left(\vec{E}_f\left(\vec{r}_{\varphi\varphi'}\right)+\vec{E}_g\left(\vec{r}_{\varphi\varphi'}\right)\right)\Big|_{z_{\varphi\varphi'}=-(h+d)}\cdot\vec{y}=\vec{E}_s\left(\vec{r}_{\varphi\varphi'}\right)\Big|_{z_{\varphi\varphi'}=-(h+d)}\cdot\vec{y} \\
\left(\vec{H}_f\left(\vec{r}_{\varphi\varphi'}\right)+\vec{H}_g\left(\vec{r}_{\varphi\varphi'}\right)\right)\Big|_{z_{\varphi\varphi'}=-(h+d)}\cdot\vec{y}=\vec{H}_s\left(\vec{r}_{\varphi\varphi'}\right)\Big|_{z_{\varphi\varphi'}=-(h+d)}\cdot\vec{y} \\
\left(\vec{E}_{inc}\left(\vec{r}_\sigma\right)+\vec{E}_1\left(\vec{r}_\sigma\right)+\vec{E}_e\left(\vec{r}_\sigma\right)\right)\cdot d\vec{l}_\sigma=0
\end{cases}
$$

sadac $\varphi=1,2,...,Q$, $\varphi'=1,2,...,P$ $\sigma=1,2,...,N$. am sistemis amoxsna kompiuteruli modelirebiT xdeba. amis Semdeg SegviZlia vipovoT difraqciis Sedegad miRebuli veli sivrcis nebismier wertilSi.

$3.3 ricxviTi eqsperimentebis Sedegebi

algoriTmis cdomilebis dadgena. sanam gadavalT konkretuli struqturebis gamokvlevaze Seqmnili programuli paketis saSualebiT, pirvel rigSi unda Semowmebul iqnas misi sizuste. es gulisxmobs damxmare parametrebis optimaluri mniSvnelobebis dadgenas. rogorc lTavSi iyo naxsenebi, damxmare parametrebs warmoadgenen: 1. damxmare zedapirebis d/λ_0 daSoreba dieleqtrikis realur zedapiridan, 2. damxmare wyaroebis raodenoba talRis sigrZis kvadratis farTobze, 3. mesris elementis mavTulis radiusi dr_0/λ, 4. elementaruli segmentebis raodenoba talRis sigrZeze. aq λ_0 da λ, Sesabamisad, warmoadgenen talRis sigrZes Tavisufal sivrceSi da dieleqtrikis SigniT. zustad am damxmare parametrebis mniSvnelobebzea damokidebuli miRebuli Sedegebis samarTlianoba. Cven vTvliT, rom Sedegi aris samarTliani, Tu sruli cdomileba sasazRvro pirobebis SesrulebaSi ar aRemateba 15% rogorc dieleqtrikis zedapirze, aseve mesris gamtari elementebis gaswvriv. unda aRiniSnos, rom am SemTxvevaSi sasazRvro pirobebis Sesruleba mowmdeba rogorc TviT kolokaciis wertilebSi, aseve maT Sua wertilebSi, sadac gadaxra am pirobebis Sesrulebidan maqsimaluria. amasTanave, ganxilul SemTxvevaSi unda srul debodes aseve kidev erTi aucilebeli piroba: Tu dieleqtriks ar gaaCnia danakargebi, dacemuli velis energia unda udrides arekvlili da gasuli velebis energiebis jams $W_{inc}=W_R+W_T$ - rasac energiis Senaxvis kanoni moiTxovs. Tu gaviTvaliswinebT, rom energia aris velis amplitudis kvadratis proporciuli, maSin SegviZlia davweroT $E_{inc}^2=E_R^2+E_T^2$, saidanac gamomdinareobs toloba $R+T=1$, sadac $R=E_R^2/E_{inc}^2$, $T=E_T^2/E_{inc}^2$ - arekvlis da gasvlis koeficientebia.

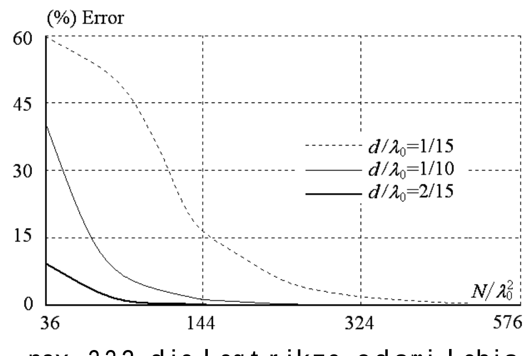
magaliTisTvis ganxilul iqna SemTxveva, rodesac usasrulo orperioduli meseri imyofeba brtyeli dieleqtrikuli fenis SigniT (nax. 3.1.1). mesris elementi warmoadgens kasinis ovals (nax. 3.3.1). dacemuli brtyeli talRa OZ RerZis sawinaaRmdego mimarTulebiT vrceldeba da gaaCnia OX polarizacia. dieleqtrikuli fenis sisqea $l=(2/3)\lambda_0$, SeRwevadobebia $\varepsilon=4$, $\mu=1$, mesris periodebia $d_1=d_2=(1/3)\lambda_0=(2/3)\lambda$.



$$\rho(\varphi)=\sqrt{c^2\cos 2\varphi+\sqrt{a^4-c^4\sin^2 2\varphi}},\ \varphi\in[0,2\pi],\ a=1.1c$$

nax. 3.3.1 kasinis ovali

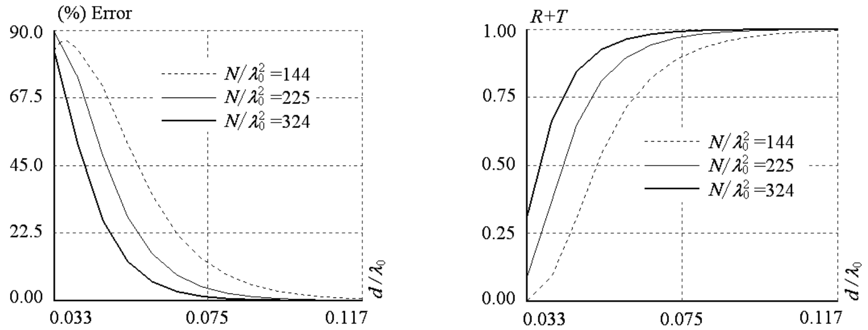
Semdeg naxazze 3.3.2 moyvanilia dieleqtrikze kolokaciis wertilebs Soris sasazRvro pirobis Sesrulebis cdomilebis damokidebuleba wertilebis raodenobaze.



nax. 3.3.2 dieleqtrikze cdomilebis damokidebuleba wertilebis raodenobaze

damxmare zedapirebi aq arian daSorebulebi realur zedapiridan sxvadasxva manZilebiT ($d/\lambda_0=1/15$, $1/10$, $2/15$). rogorc vxedavT 36 kolokaciis wertili talRis sigrZis kvadratis farTobze maRal cdomilebas iZleva, rodesac $d/\lambda_0=1/15$, Tumca maTi raodenobis gazrdiT cdomileba mkveTrad mcirdeba da 324 wertilis SemTxvevaSi (18 wertili t a l Ris sigrZeze) mas ukve dasaSvebi mniSvne l oba gaaCnia. me t i daSo reba kidev u f r o amc i rebs c domi l ebas, maga l iTad, Tu $d/\lambda_0 = 2/15$ maSin c domi l eba 144 we r t i l is SemTxvevaSi (12 we r t i l i t a l Ris sigrZeze) mxo l od 2%-s Seadgens.

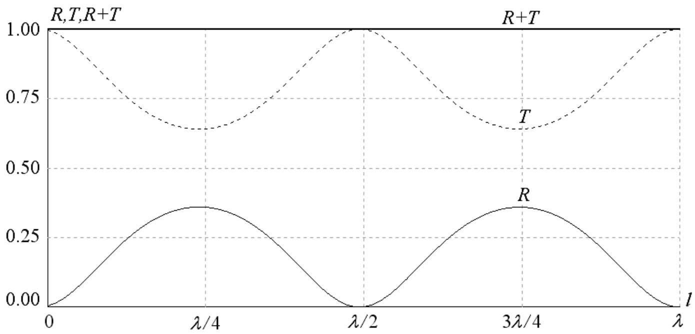
Semdeg moyvani l ia igive c domi l ebis damokidebu l eba damxma r e zedapi rebis d daSo rebaze die l eqt r ikis zedapi r idan (nax. 3.3.3 a)). daSo rebas ganicdian r ogo r c Sida, aseve ga r e damxma r e zedapi rebi. aseve, pa r a l e l u r ad, moyvani l ia ene r giis ba l ansis g r a f iki (nax. 3.3.3 b)).

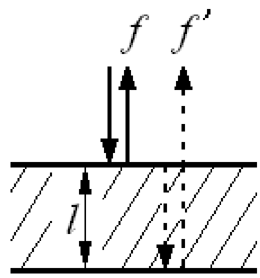


a) b)

nax. 3.3.3 a) die l eqt r ikze c domi l ebis da b) ene r giis ba l ansis pi r obis damokidebu l eba damxma r e zedapi r ebis daSo r ebaze

Semdeg naxazze moyvani l ia R a r ekv l is, T gasv l is koe f i c ienebis da aseve maTi $R+T$ j amis damokidebu l eba die l eqt r ikis l sisqeze (nax. 3.3.4). r ogo r c vxedavT, a r ekv l is da gasv l is koe f i c ientebi pe r iodu l ad i cv l ebian. es aixsneba imiT, r om die l eqt r ikis sisqis cv l i l ebis d r os, i cv l eba fazaTa sxvaoba zeda zedapi r idan a r ekv l i l f ve l sa da qveda zedapi r idan a r ekv l i l s da Semdeg gamosu l f' ve l ebs So r is (nax. 3.3.5).





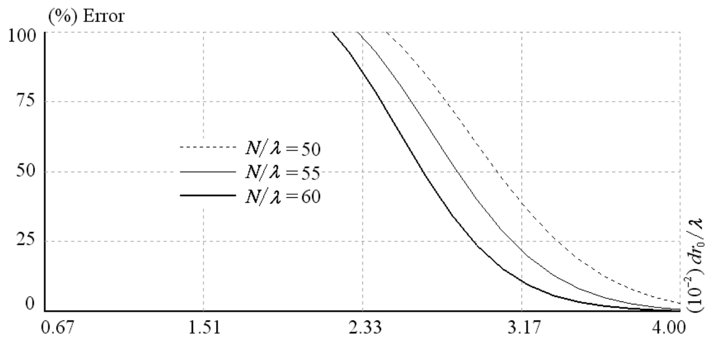
nax. 3.3.4 R, T koe f i c ientebis da maTi j amis damokidebu l eba die l eqt r ikis sisqeze

nax. 3.3.5 f da f' ve l ebi

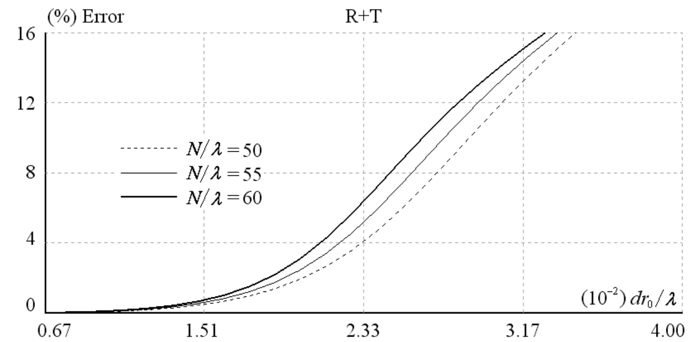
amis Sedegad, zogie r T SemTxvevaSi (a r ekv l is koe f i c ient is maqsimumebi) adgi l i aqvs maTi fazebis damTxvevas, rac, inte r fe r enciis Sedegad iwvevs a r ekv l i l i ve l is gaZ l ie r ebas. sxva SemTxvevaSi (a r ekv l is koe f i c ient is minimumebi) f da f' ve l ebi sawinaaRmdego fazebiT xvdebian da faqt iu r ad axSoben e r TmaneTs, rac Sesabamisad iwvevs gasv l is koe f i c ient is gaz r das. ami t om g r a f ikis pe r iodu l oba a r is $\lambda/2$. nebismie r SemTxvevaSi, a r ekv l is da gasv l is koe f i c ientebis j ami e r Tis t o l ia, rac TanxmobaSia ene r giis

Senaxvis kanonTan. es miRebuli Sedegi aseve adasturebs kompiuteruli algoriTmis samarTlianobas.

mesris elementze cdomilebis angariSis dros ganxilul iqna rezonansuli SemTxveva, rodesac mesris elementis sruli sigrZe faqtiurad talRis sigrZis tolia dieleqtrikis SigniT ($L = 1.03\lambda$). gasagebia rom cdomileba sxva (ararezonansul) SemTxvevaSi miRebulze mcire unda iyos. Catarebulma kvlevebma gviCvena rom mavTulis radiusis gazrdiT mcirdeba sasazRvro pirobis Sesrulebis cdomileba, magram am SemTxvevaSi mcired irRveva $R+T=1$ toloba. naTqvami naTlad Cans momdevno ori grafikidan, sadac moyvanilia cdomileba sasazRvro pirobisaTvis da $R+T=1$ tolobisaTvis.



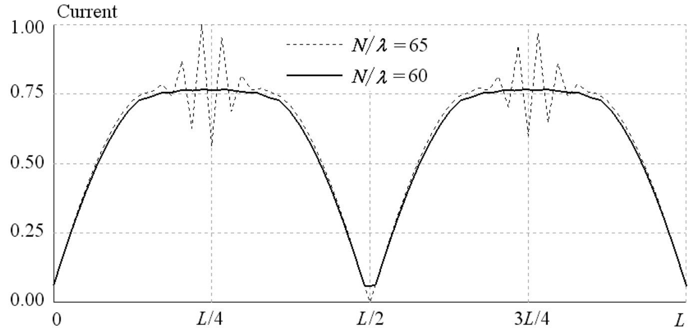
a)



b)

nax. 3.3.6 a) mesris elementze sasazRvro pirobis da b) energiis balansis cdomilebebis damokidebuleba mavTulis radiusze

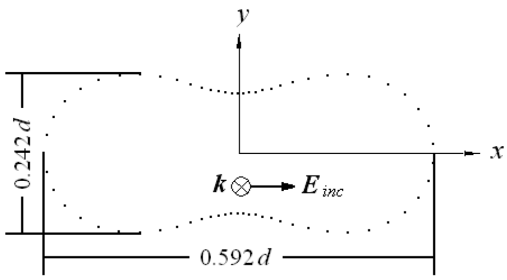
am suraTebze moyvanilia sami grafiki, romelnic Seesabamebian mesris elementze kolokaciis wertilebis sxvadasxva raodenobas. rogorc vxedavT, cdomileba orive SemTxvevaSi SedarebiT mcirdeba rodesac wertilebis aRniSnuli raodenoba izrdeba, Tumca igi ar unda aRematebodes 60-s talRis sigrZeze, radgan winaaRmdeg SemTxvevaSi vRebulobT arafizikur denebis ganawilebas elementSi. es kargad Cans Semdeg suraTze, sadac moyvanilia maqsimalur mniSvnelobaze danormirebuli inducirebuli denis ganawileba elementSi (nax. 3.3.7).



nax. 3.3.7 danormirebuli inducirebuli
denis ganawileba mesris elementSi

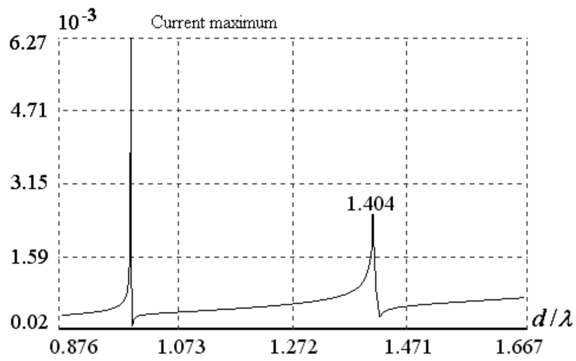
maSasadame, miRebuli mrudebis Tanaxmad, rezonansis SemTxvevaSi, fizikuri Sedegebis misaRebad saWiroa parametrebis Semdegi mniSvnelobebi: $N/\lambda = 60$ $3.17 \cdot 10^{-2} < dr_0/\lambda < 4 \cdot 10^{-2}$. unda aRiniSnos, rom mesris elementebis dayofa tol monakveTebad zogad SemTxvevaSi kasinis elementebisaTvis rTuldeba da talRis sigrZeze elementis gaswvriv kolokaciis wertilebis aseTi didi raodenoba nawilobriv amiT aixsneba.

Tavisufali sivrcis SemTxveva. magaliTisTvis ganxilul iqna kerZo SemTxveva, rodesac $l = 2d$ sisqis brtyeli dieleqtrikuli fenis SeRwevadobebia $\varepsilon = \mu = 1$, rac Tavisufal sivrces Seesabameba. mesris periodebia $d_1 = d_2 = d$. damxmare zedapirebis daSoreba udris $0.4d$, xolo mavTulis sisqea $0.02d$. Semdeg suraTze moyvanilia kasinis elementis geometria da dacemuli velis orientacia (nax. 3.3.8). mas gaaCnia erTeulovani amplituda da OX polarizacia.



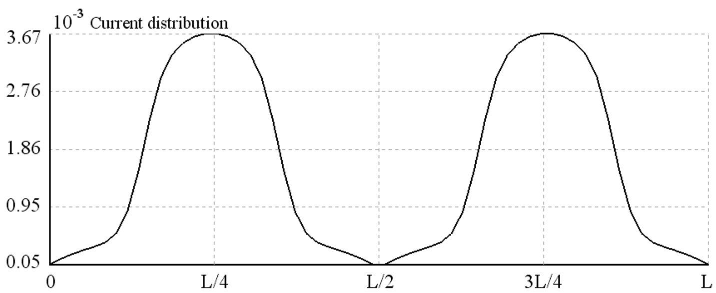
nax. 3.3.8 elementis geometria da
dacemuli talRis orientacia

dacemuli veilis talRis sigrZis cvlilebiT Seswavlilia elementSi inducirebuli denis maqsimumis cvlileba. naxazze 3.3.9 moyvanil diapazonSi, napovnia ganxiluli sistemis parametrebis ori mniSvneloba, rodesac elementSi aRZrul dens maRali mniSvneloba gaaCnia: $d/\lambda = 1$ da $d/\lambda = 1.404$. pirveli maqsimumis mniSvneloba warmoadgens mxolod mesris rezonanss. meore maqsimumis mniSvneloba Seesabameba ormagi rezonansis SemTxvevas (rezonanSia TviT elementi da aseve rezonansulia mesris periodi). rogorc vxedavT, aRZruli deni rezonansebis SemTxvevaSi ori rigiT metia.

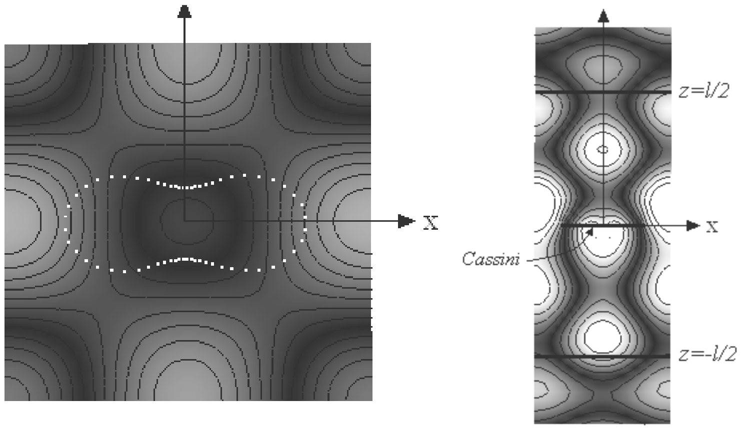


nax. 3.3.9 denis maqsimumis damokidebuleba
dacemuli talRis sigrZeze

Semdeg suraTze (nax. 3.3.10) moyvanilia denis ganawileba mesris elementSi rodesac $d/\lambda = 1.404$. rogorc vxedavT, OX polarizaciis SemTxvevaSi elementis Cazneqil nawilSi ufro maRali deni aRiZvreba.



nax. 3.3.10 denis ganawileba elementSi
ormagi rezonansis dros



a) b)

nax. 3.3.11 axlo velis ganawileba a) mesris
paralelur da b) marTobul sibrtyeebSi

naxazze 3.3.11 moyvanilia axlo velis E_x komponentis ganawileba mesris periodis farglebSi mis paralelur da marTobul sibrtyeSi, ormagi rezonansis SemTxvevaSi, rodesac $d/\lambda = 1.404$. velis daxatvis sibrtye a) suraTze imyofeba mesris sibrtyidan 0.3λ simaRleze.

aRsaniSnavia, rom dieleqtrikis zedapirebze $z=\pm l/2$ veli kargad ikereba rodesac $\varepsilon=1$.

daskvna

amoxsnil iqna brtyeli talRis difraqciis amocana sistemaze usasrulo orperioduli meseri - brtyeli dieleqtrikuli fena. ganxilul iqna SemTxvevebi, rodesac meseri imyofeba fenis SigniT da aseve mis gareT. es amocana pirvel SemTxvevaSi amoxsnilia damxmare gamomsxiveblebis meTodiT, romelSic orperioduli grinis funqcia asrulebda damxmare gamomsxivebelis velis rols. meore SemTxvevaSi am ricxviTi meTodis garda, ganxilul iqna aseve amoxsnis ori mkacri meTodi. Semdeg iqna moyvanili ricxviTi gamoTvlebis Sedegebi, sadac pirvel rigSi dadginda damxmare parametrebis optimaluri mniSvnelobebi. magaliTisaTvis ganxilul iqna kasinis elementebisagan Semdgari meseri.

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